ROWAN UNIVERSITY
Department of Chemistry and Physics

Experiments in Physics

Fifth Edition

Experiments in Physics

A Laboratory Manual for Introductory Physics

Fifth Edition

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1. Introduction to Motion: Distance-Time and Velocity-Time Graphs

Introduction

In this lab you will measure your motion with a sonic motion detector. The motion detector emits sound waves and then detects them after they have reflected off of some object (such as you or your lab partners!). A computer then records distance from the detector to the object as a function of time. You will practice making and interpreting graphs.

Equipment

• motion detector • ULI computer interface • MacMotion program • 2 meter stick

Before the Lab

Read the sections in your text describing one dimensional motion. Be familiar with the concepts of position, velocity, acceleration and the slope of a graph.

Procedure

Setting up the motion detector and MacMotion program

1) Open the MacMotion program. The motion detector uses the ULI (Universal Lab Interface). Be sure the ULI is connected to the computer and turned on and that the motion detector is plugged in.

Click on the START button (on the screen) to make a graph of distance versus time. The electronics will make an audible "clicking" sound when the detector is working and the distance from the detector to an object will be plotted. (Note that the sound you hear is not the sound used to detect motion, which is inaudible, but rather the electronics in the detector clicking.)

Practice with the detector until you understand the reading. Sound from the detector is emitted in a cone that spreads out from the motion detector. If the sound is reflected by a fixed object (such as a wall) a straight line will appear on the graph since distance to the wall is not changing as a function of time. Position your detector so it does not hit any fixed objects and you are able to walk back and forth in front of it. Move back and forth in front of the detector and watch the distance graph change.

Calibrate the distance from your detector. Stand 0.5 meters, 1 meter and 2 meters from the detector and make sure that the distances recorded correspond to your position.

For your report

Label each part of your report with the appropriate section numbers, corresponding to the hand-out.

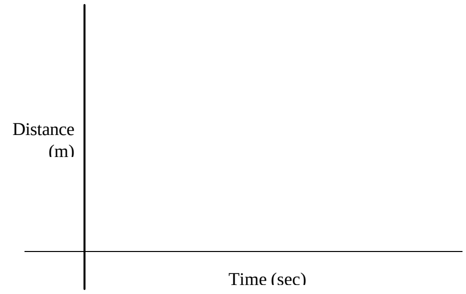
Label the axes on your graphs with numbers and units.

When discussing your graphs you can label points on them ((A), (B), (C), *etc.*) in order to refer to them in your discussion.

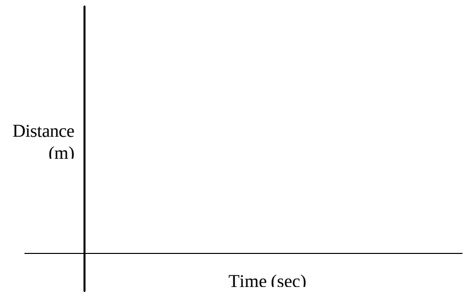
Answer questions in complete sentences so that the answers can be understood independently of the handout.

Activity 1: Make distance-time graphs for different walking speeds and directions

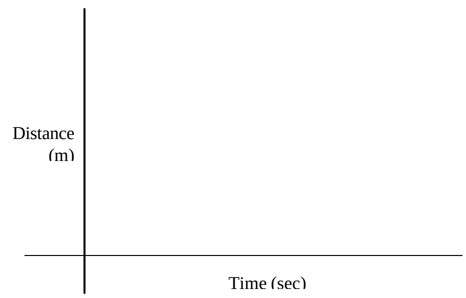
Start at the ½ meter mark and make a distance versus time graph, walking *away from* the detector *slowly and steadily* (at constant speed). Sketch the graph.



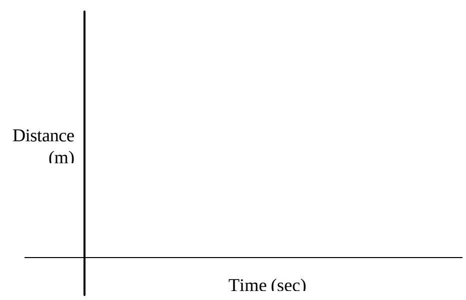
Make a distance-time graph walking *away from* the detector *medium fast and steadily*. Sketch the graph.



Make a distance-time graph walking *toward* the detector *slowly and steadily*. Sketch the graph.



Make a distance time graph walking *toward* the detector *fast and steadily*. Sketch the graph.



Questions:

Answer the following. Consider using these words or phrases (as well as your own): up, down, steeper, less steep, rising, falling, increasing, decreasing, faster, slower.

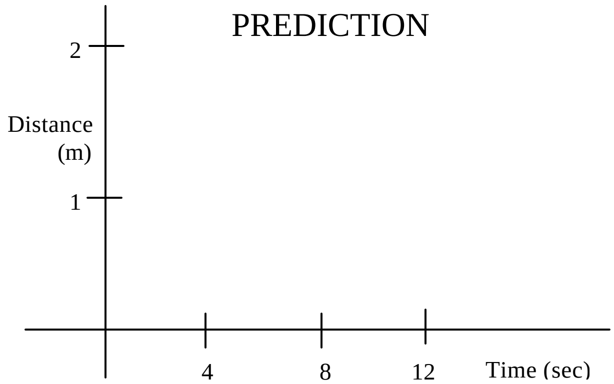
How is distance changing as a function of time when you walk *away* from the detector. When you walk *toward* the detector?

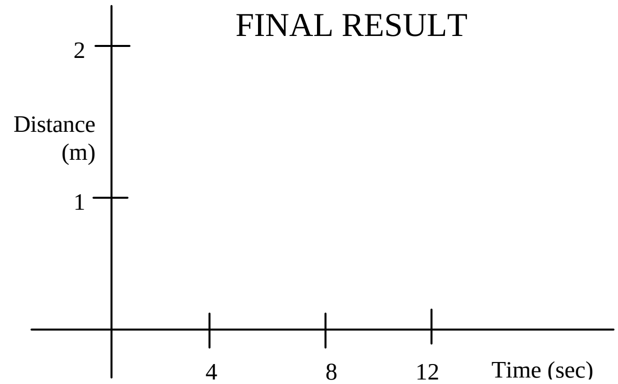
How does you graph show whether you are moving away from the detector or towards the detector?

How does your graph show whether you are moving quickly or slowly?

Activity 2: Prediction

Make a prediction of how a distance-time graph would look for a person starting at the 1 meter mark, walking *away slowly and steadily* for 4 seconds, stop for 4 seconds, then walk *toward* the detector *more quickly*. Draw your prediction using a dotted line. Compare predictions within your group and discuss them. Arrive at a final prediction and draw it with a solid line (do not erase your original prediction).





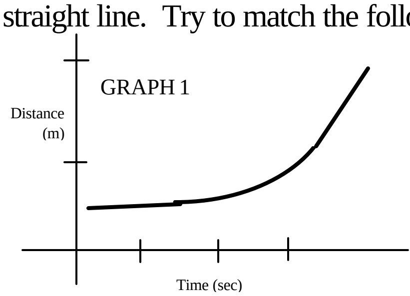
Do the experiment Move in the way described and graph your motion. When you are satisfied with the experiment, draw your groups final result.

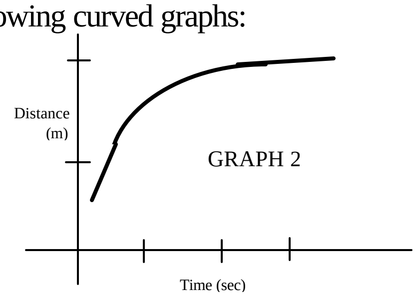
Question

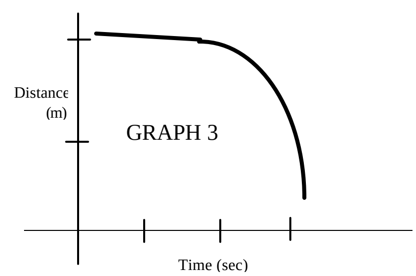
Is your prediction the same as the final result? How does the graph show when you are moving away or towards? The speed of motion? Stationary motion? If the results don't agree how would you change the way you move?

Activity 3: Moving at non-constant speed

The *slope* of the graph shows the speed of motion. When speed is constant the graph is a straight line. Try to match the following curved graphs:







Questions

For each graph describe how you had to move to match the graph. Use word like faster, slower, going toward, going away, increasing speed, decreasing speed, steeper, less steep.

How does a graph where the speed is changing differ from a graph where you are moving *steadily* (at constant speed)?

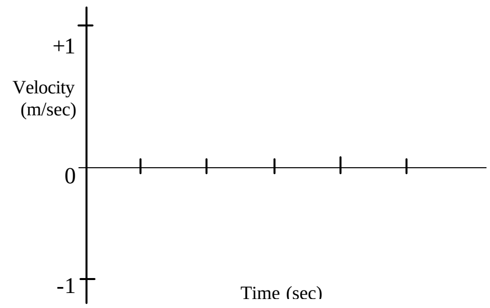
Acceleration is the rate of change of speed. When speed is constant acceleration is zero (no change means the rate of change is zero). How does the *slope* of the graph change when there is acceleration?

Activity 4: Velocity-time graphs

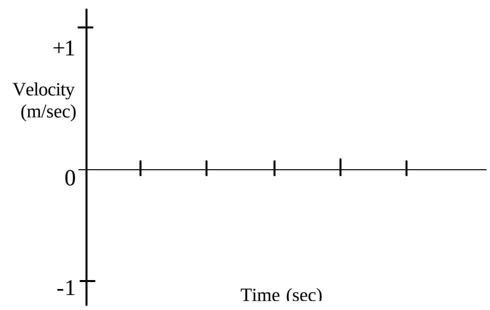
Set up the MacMotion program to display velocity. Double click on the distance graph and select **Velocity** in the dialog box. Set the range from -1 to 1 meters/sec. Change the Time scale to read from 0 to 5 sec.

Graph your velocity for moving different speeds and directions:

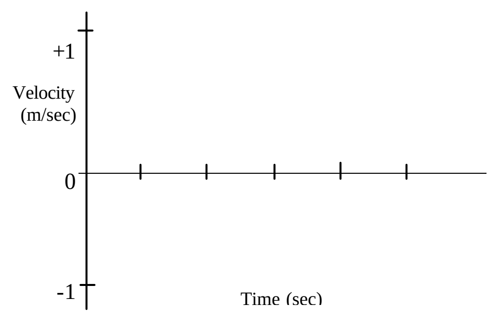
Make velocity-time graph, walking *away from* the detector *slowly and steadily*. Sketch the graph.



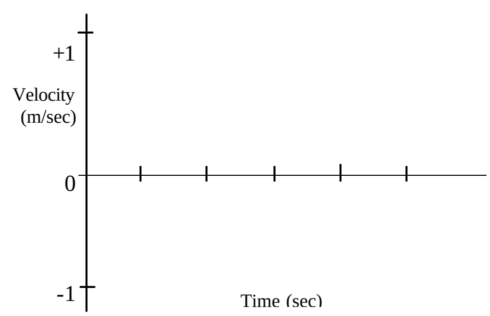
Make a velocity versus time graph walking *toward* the detector *medium fast and steadily*. Sketch the graph.



Make a velocity versus time graph by first walking *fast and steadily away from the detector,* then *pausing* for about a second, then walking *slowly toward the detector*



Make a velocity graph first standing still for 1 second, then walking *toward* the detector first slowly and then with increasing speed.



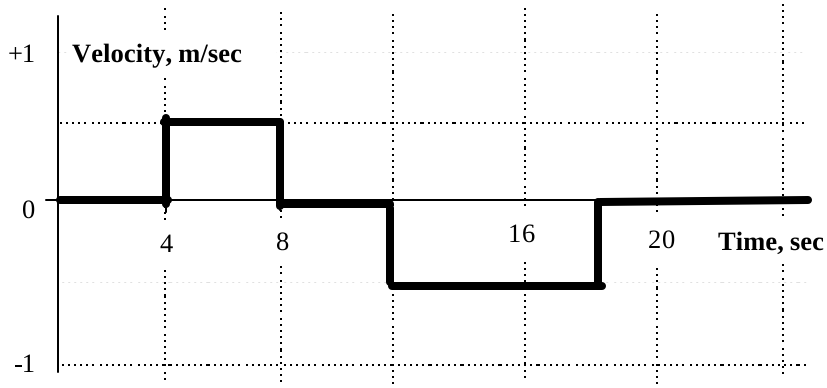
Questions

How do the plus and minus signs of the velocity show the *direction* of motion.

Does the sign of the velocity show whether the distance is *increasing* or *decreasing* as a function of time? Explain.

Activity 5: Match a Velocity graph

Pull down the **File** menu and select **Open**. Double click on **Velocity.Match.** The velocity graph will appear on the screen: Try to create a matching graph.. You may may need to tray a number of times. Work as a team and plan your movements so as to match the times and velocities.



Questions

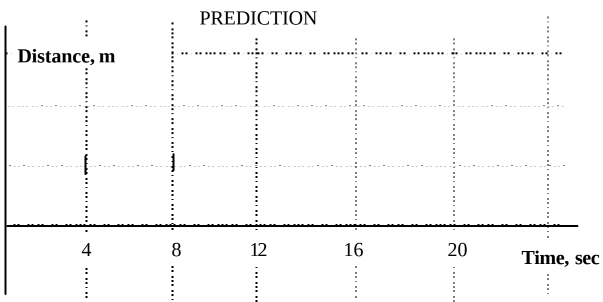
Sketch your graph. Referring to different points on your sketch (you might label them A, B, C, D etc.) describe how you move to match the graph.

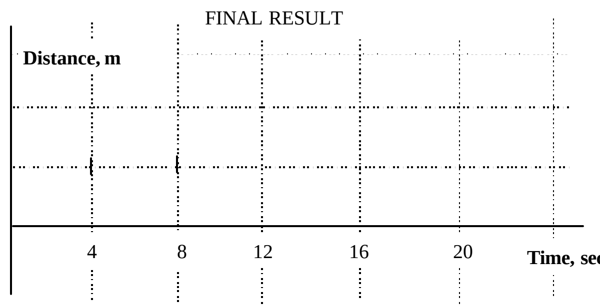
Is it possible to move so as to create a perfectly vertical line on the velocity graph? Explain.

Activity 6: Making a corresponding distance graph

Make a prediction of how a distance-time graph would look for the motion you have just made. It may help to have one lab partner repeat the motion as you discuss how the corresponding graph should look. Make a sketch of your prediction.

Switch the program back to the Distance-time graph and repeat your motions. Sketch the resulting graph next to your prediction.





Questions

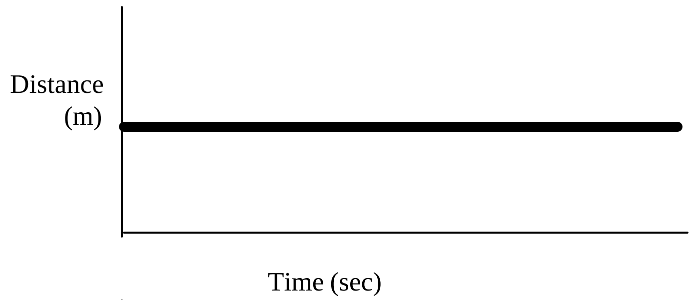
What parts of the velocity graph correspond to a horizontal line on the distance graph?

What parts of the velocity graph correspond to increasing distance? What does the *slope* of the distance graph look like in that case?

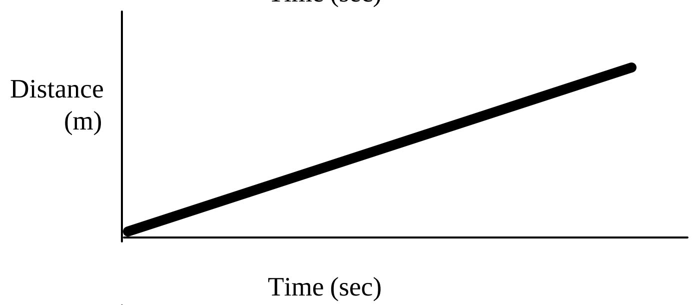
What parts of the velocity graph correspond to decreasing distance? What does the *slope* of the distance graph look like in that case?

Supplemental Questions:

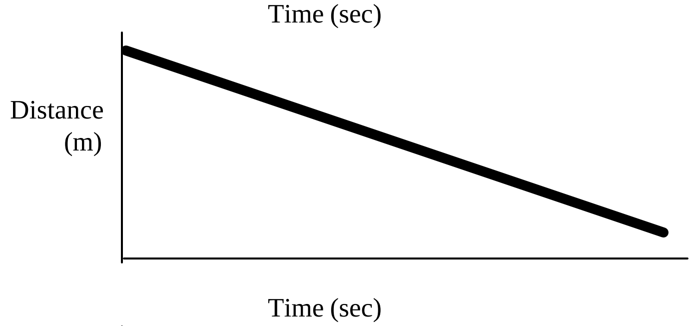
1. What do you do to create a horizontal line on the distance versus time graph?



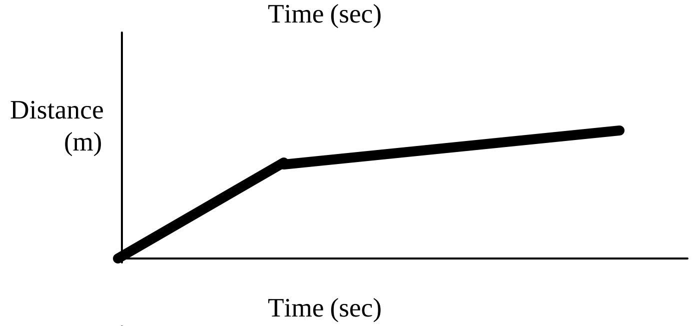
2. How do you walk to create a straight line that slopes up?



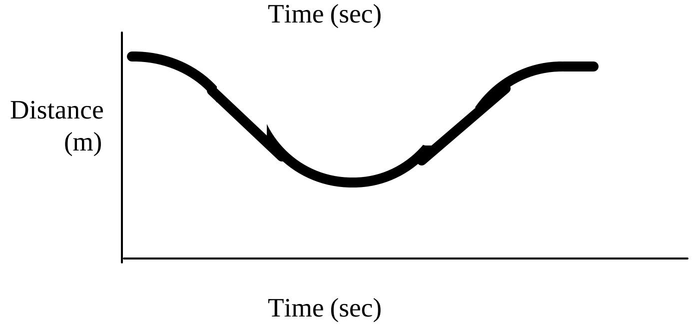
3. How do you walk to create a straight line that slopes down?



4. How do you move to create a line which goes up steeply at first, then goes up less steeply?

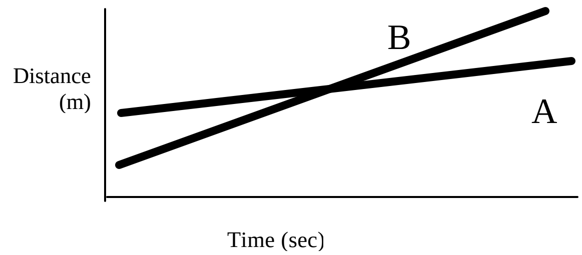


How do you move to create a U-shaped graph?



For which parts of the U-shaped graph is the velocity zero?

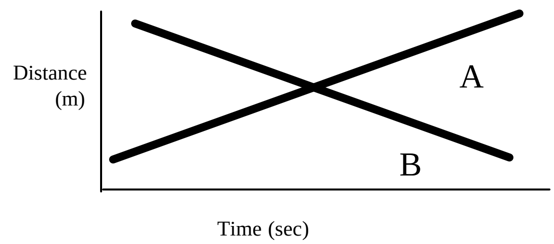
6. Which object is moving faster, A or B?



Which starts in front? Which finishes in front?

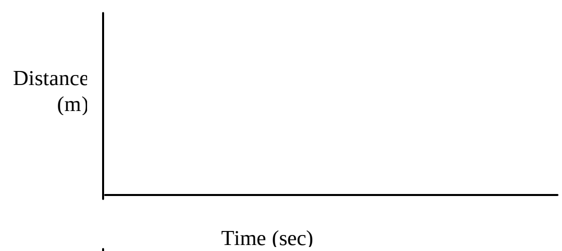
What happens at the intersection of the two lines?

7. Which object is moving faster (can you tell?)

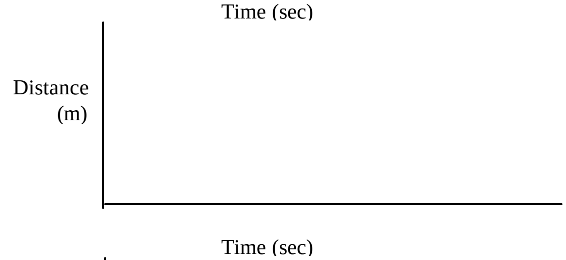


Which is moving towards the detector? Which away?

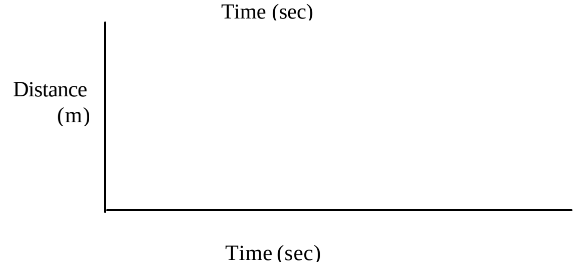
8. Sketch a graph for an object moving steadily away from the detector.



9. Sketch a graph for an object which is standing still..



10. Sketch a graph for an object moving quickly towards the detector and then slows down and stops



2. Gravitational Acceleration: A case of constant acceleration

Introduction

The gravitational force is one of the fundamental forces of nature. Under the influence of this force all objects with non-zero mass are attracted by the mass of other objects. This means that near the surface of the earth all objects are attracted by the mass of the earth itself and will fall towards the center of the earth under the influence of gravity. Because the attraction is proportional to mass, all objects will fall with the same constant acceleration, which is denoted by the letter *g*.

Equipment

- free-fall apparatus, voltage supply
- paper to record spark-tracks
- ruler, graphing software

Note: be sure and ground the free fall aparatus when connecting it to the power supply

Before the Lab

Read the sections in your text describing one dimensional motion and constant acceleration. Be familiar with the concepts of position, velocity, acceleration and the slope of a graph.

Theory

An object falling near the surface of the earth experiences a uniform (constant) acceleration. Because acceleration is constant, in a downward direction, and equal to g, its motion can be described by the equations:

$$y(t) = y_0 + v_0 t + \tfrac{1}{2} g t^2$$
$$v(t) = v_0 + gt$$

Here we have defined "down" as being the positive y direction.

These equations predict that a plot of <u>velocity</u> versus time will start at the value v_o at time t=0 and show a straight line with negative slope. The value of that slope will be equal to acceleration (the rate of *change* of velocity, $a=dv/dt$). Thus we can measure the value of g be determining that slope of a graph which plots velocity versus time for a falling object.

Procedure

In this lab you will measure the free-fall acceleration of a mass. The free-fall apparatus consist of a metal object which falls down a track between two wires. A voltage pulse between the two wires causes sparks to jump from the falling mass to one of the wires. Paper is placed between the falling object and this wire so that the spark makes a mark on the paper. This records the position of the object as a function of time. The voltage pulses are applied 60 times per second, so the time between pulses is $1/60$ seconds.

Your lab instructor will make a tape for you or assist you in making one. Carefully examine the tapes and draw small circles around the spark marks so their locations are easily seen. (Do not cover the marks themselves, just outline them). Occasionally a spot in the sequence may be missing. If you think there is a missing spot, draw a question mark about where you think it should be.

Choose one spot near the top to be "point 0" where the position is y_o=0 at time t=0. This is not the first spot. Do not choose the large spot where the object rested prior to being released! Choose a spot which is easy to measure and where all the following points appear to travel in a straight line. Because this is not the "first" spot we cannot not assume that the initial velocity, v_o, is equal to zero.

Using the data table (at the end of this handout) begin numbering points 0,1,2,3… and record times for each point so that point 0 is t_0=0, point 1 is t_1=1/60 sec=0.0167 sec, point 2 is t_2=2/60 sec =0.0333 sec etc. Wherever there are missing spots you should include a position for them in your table and write "missing" beside these points.

Draw a straight line through each spot perpendicular to the edge of the tape. You will use these lines to measure the distance to each spot.

Carefully measure the distance from spot number zero to each following spot on your tape. Record these positions in your table as position y_i.

You will now compute the velocity at for each spot. At point " *i* " (where *i* means 1,2,3 etc.) the position is given by:

$$y_i = y_0 + v_0 t_i + \frac{1}{2} g t_i^{\,2} \qquad \text{(Equations 1)}$$
$$v_i = v_0 + g t_i$$

It can also be shown for constant acceleration only that the average velocity between two times is equal to the instantaneous velocity at a time halfway in between. Thus, for equally spaced time intervals, the instantaneous velocity at time t_2 is equal to the average velocities between times t_1 and t_3:

$$v_2 = \bar{v} = \frac{y_3 - y_1}{t_3 - t_1} \qquad \text{(Equation 2)}$$

Use Equation 2 to find the instantaneous velocity for points 1, 2, 3 etc. Use the positions and times for points 3 and 1 to calculate v_2, for points 2 and 4 to calculate v_3, for points 3 and 5 to calculate v_4, etc. Record each in your data table.

You can use a similar equation to calculate and record average accelerations:

$$a_2 = \bar{a} = \frac{v_3 - v_1}{t_3 - t_1} \qquad \text{(Equation 3)}$$

Make a graph of velocity versus time for your data. Your instructor will show you how to do this on the computer. If your graph makes it apparent that you failed to take into account missing points, adjust the times accordingly. See appendix for notes on using the computer programs.

Again using the computer, fit a straight line to your data. The slope of this line will be your experimental value for *g*, which you can call $g_{experimental}$.

Determine the slope of your line and record it with your data.

Calculate and record the percent difference between the experimental value and the theoretical value, *g*=9.8 m/sec^2 using the equation:

$$\text{Percent difference} = \Delta g = \frac{g_{experimental} - g}{g} \times 100\% \qquad \text{(Equation 4)}$$

If v_o is in fact not zero at t=0, then your graph will intercept the vertical axis at some velocity other than zero. From your graph determine v_o.

Make a plot of position versus time using the computer. If position increases as time squared then this will be a *quadratic* curve. Use the quadratic curve fit to fit this data and find a second value for "g".

For your report

Print copies of the graphs that you made for each student in your group. On each graph you should label the axis and record the slope and intercept. Write a cover page with a summary describing the experiment. Explain how you obtained the values of g and v_0 from your data. Compare the value for g to the accepted value and discuss possible sources of discrepancy. *You must explain why you expect your velocity graph to be a straight line and why its slope should be "g".*

Comment on how the value of "g" determined from your graphs compares to the variation in values calulated point-by-point in your table.

Suppose the time difference between sparks were not precisely 1/60 second. How would that affect your results? (Would $g_{\text{experimental}}$ be too large? too small? the same?

Data Table: Free Fall with Spark Timer

Point i	Time from t=0, t_i	Distance from y=0, y_i	Calculated velocity, v_i	Calculated acceleration, a_i
0	0 sec	0 cm		
1				
2				

Remember measurements should include units!

Value of "g" from velocity graph, $g_{\text{experimental}}$ _________________________

Percent difference from accepted value, $\%\Delta g$ _________________________

Initial speed from velocity graph, v_o _________________________

Value of "g" from position graph, $g_{\text{experimental}}$ _________________________

Percent difference from accepted value, $\%\Delta g$ _________________________

Initial speed from position graph, v_o _________________________

3. The Force Table: Addition and Resolution of Vectors

Introduction

Many of the concepts used in physics must be described by both a *magnitude*, or size, and a *direction*. Some of these quantities are displacement, velocity, force, and acceleration. We use a *vector* to represent these quantities. A vector can be represented graphically by an arrow whose length is proportional to its magnitude and which points in the desired direction. It can also be represented mathematically by giving the *components* of the vector along three perpendicular directions. In this experiment we will investigate methods of adding the vectors that represent forces. We will practice resolving the vectors into components. We will use a force table to experimentally observe the addition of different force vectors.

Equipment

- Force Table with four pulleys
- Four weight hangers
- Set of slotted weights
- String
- Protractor, Ruler
- Graph Paper

Before the Lab

Read the sections in your text describing vectors and how to add them. You should learn how to add vectors both *graphically* (by constructing a triangle or parallelogram) and by resolving them into perpendicular *components* and then adding the components.

Theory

Vector Components:

When we wish to describe a quantity which has both a magnitude (size) and direction, we can describe it as a *vector*. The vector can be described in terms of *components*, analogous to directions on a treasure map. To describe the direction of the treasure, we might say it is 5 strides in a northeasterly direction, or we might describe it as lying 3 strides east and 4 strides north, as shown in Figure 1. Similarly the vector **A** can be broken into components A_x and A_y. We describe this vector in terms of a horizontal direction, x, and a vertical direction, y (as shown in the figure). The angle q measures the angle from the x-axis to the vector, **A**. The relationship between the magnitude (or length) of **A** and its components are then

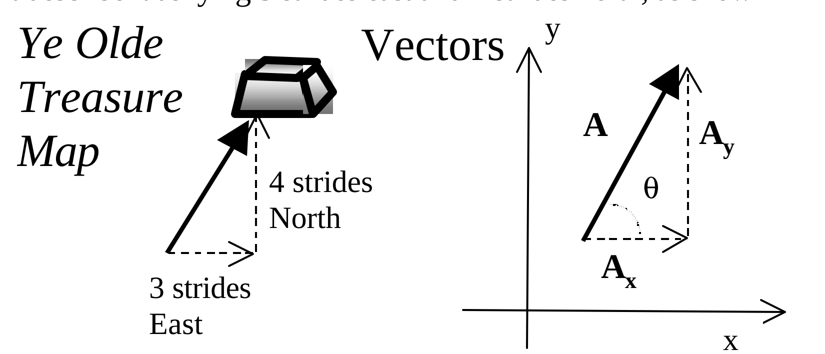


Figure 1: Components of a vector are analogous to directions on a treasure map.

$A_x = A\cos q$ where A_x is the side adjacent (or closest) to the angle

$A_y = A\sin q$ where A_y is the side opposite to the angle

For example if **A** is a vector 5 units long and the angle q is equal to 53.1 degrees then we would find that

$$A_x = 5\cos\left(53.1^\circ\right) = 3$$
$$A_y = 5\sin\left(53.1^\circ\right) = 4$$

Thus **A** might represent the distance and direction to travel to find the treasure.

We can also find the magnitude and angle associated with **A** if we know its components, since

$$A = \sqrt{A_x{}^2 + A_y{}^2} \qquad \text{and} \qquad q = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

All of these relationships follow from Pythagorus' theorem since A, A_y and A_x form a right triangle.

Adding Vectors:

When we wish to add two vectors, **A** and **B**, to form a resultant vector, **C**, then we simply add their components to find the components of **C.**

The vector **C** can be found graphically by positioning **A** and **B** head to tail, as in the left side of Figure 2. **C** will be the vector that goes from the tail of **A** to the head of **B** and will be the diagonal of the parallelogram formed by **A** and **B**. The components of the vector **C** can also be found from the components of **A** and **B**.

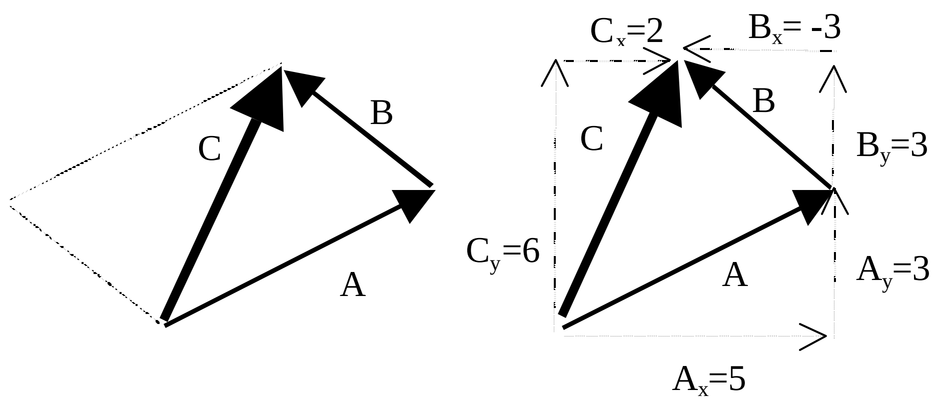


Figure 2: Addition of vectors. C is the vector sum of A and B. It can be found either graphically or by adding components.

$$\text{If} \quad \vec{C} = \vec{A} + \vec{B} \quad \text{then} \quad \begin{cases} \mathrm{C}_x = A_x + B_x \\ \mathrm{C}_y = A_y + B_y \end{cases}$$

In Figure 2, **A=** A_x ***i*** $+A_y$ ***j***=5 ***i*** +3 ***j*** and **B=** B_x ***i*** $+B_y$ ***j***= -3 ***i*** +3 ***j*** then **C**=(5-3)***i***+(3+3)***j***=2***i*** +6***j*** or in other words C_x=2 and C_y=6. (Here the symbols ***i*** and ***j*** are used to denote the x and y directions, respectively.)

Vector Practice

Find x and y components of a vector which is 10 units in length and 40 degrees below the x axis (below the x axis is -40 degrees)

A_x= ___________________ A_y= ___________________

Check your results:

$\sqrt{A_x{}^2 + A_y{}^2}$ = _________ $\tan^{-1}\left(\frac{A_y}{A_x}\right)$ = __________

2. Find the components of a vector which has a length of 7 units and a direction 20 degrees above the x axis.

B_x= ___________________ B_y= ___________________

Check your results as above.

3. Find the result of adding **A** and **B** to find the component of **C = A + B**

C_x = _____________ C_y = ______________

Make a sketch showing the vectors A, B, and C and check the addition graphically (putting A and B head to tail and drawing a parallelogram).

Experimental Procedure

The *Force Table* is an apparatus which can be used to determine the resultant (or vector sum) of different forces. Weight forces are applied to a central ring by means of strings which run over pulleys and then hang over the edge of the table. The magnitudes of the forces may be varied by adding and removing weights and their directions can be varied by moving the pulleys. When two or more weight forces are applied to the ring, their vector sum, or resultant, can be found by finding the additional force needed to exactly balance the applied force. For example, if two forces are applied, the resultant, or vector sum, is

$$\vec{F}_1 + \vec{F}_2 = \vec{F}_{total}$$

the magnitude and direction of $\vec{F}_{total}$ may be found by finding a third force, $\vec{F}_3$, such that

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

When the net force on the ring is zero it will remain centered on the table in *equilibrium*. The sum of $\vec{F}_1$ and $\vec{F}_2$ must then be equal in magnitude, but opposite in direction, to $\vec{F}_3$, i.e.,

$$\vec{F}_1 + \vec{F}_2 = -\vec{F}_3$$

In the following experiments you will practice different methods of adding vectors and then use the force table to experimentally check your calculations.

Begin by leveling the force table, if necessary. Then practice balancing forces until you are able to determine when there is zero net force on the ring. Note that it is important that the strings tied to the ring slide easily from side to side, so that no sideways force is applied to the ring. The strings should pull straight outwards toward positions of the pulleys on the edges of the table. There will be some *uncertainty* in your experimental method. <u>Remember that the weight of the hangers must be included in your total weights.</u>

Given two force vectors, for example $\mathbf{F}_1$ corresponding to 200 gm at 30^0 (above the positive x-axis) and $\mathbf{F}_2$ corresponding to 300 gm at 140^0, find their resultant, or vector sum by these three methods. Record your results in your lab notebook, drawing appropriate diagrams to describe both your calculations and your experimental method.

Graphically: draw a diagram to scale and construct a parallelogram to find the sum.

Addition of components: on your diagram, define an x- and y-axis. Resolve the vectors into components along these axis (with your calculator) and find the sum. Does your answer make sense when you compare the result with the graphical method?

Experimentally: Use the force table to determine the third force that would be required to balance the two force vectors previously defined. This force should be 180^0 opposite the force found by the previous two methods (since it cancels the first two forces). How does the result compare to you calculations. What is the *uncertainty* in your experimental result?

Repeat with two different forces in different directions, for example 200 gm at 20^0 and 150 gm at -80^0. This time try the experimental method first. Set up your two forces, then pull on the string to find the direction that a third force must be applied to balance the first two. Add weights to determine the third force. Then check your result by graphical methods and by addition of components. How do your results compare? Is this within the expected uncertainty of the experimental method?

Repeat using two perpendicular forces. You can choose the x- and y-axis to be in the perpendicular directions. for example $\mathbf{F}_1=F_x$ corresponding to 150 gm at 0^0 and $\mathbf{F}_2=F_y$ corresponding to 200 gm at 90^0.

You can also find the components of a vector experimentally. Place three pulleys at 240^0, 90^0 and 0^0. Hang a total of about 300 gm from the string through the pulley at 240^0. You should now be able to find its components along the x-axis (at 0 degrees) and the y-axis (at 90 degrees) by finding the weights that you must hang from these pulleys to balance the weights. Check your results graphically and mathematically.

Questions

Compare the graphical and analytical (addition of components) methods for adding vectors. Which is more accurate? Give possible sources of error for both methods. Why is it useful to use both methods?

What are the possible sources of error in the experimental method? (Why is it necessary to allow the strings to slip loosely about the ring.)

If the weights of all the weight hangers were the same, could their weight have been neglected? Explain.

What is the effect of the weight of the ring? What difference would it make if the ring were considerably more massive?

4. Atwood’s Machine: Newton’s Second Law

Introduction

A physical “law” is a statement of one of the fundamental theoretical principles that underlie our understanding of how the physical world works. Newton’s First and Second Laws describe the relationship between force, mass, and acceleration. The net (total) force on an object, which is the vector sum of all applied forces, is equal to the total mass times the acceleration of the object. In this lab you will use a simple system of pulleys, strings and weights to investigate this relationship.

Equipment

• Pulley Level	• String, mass hangers	• 2 Meter stick
• clamps and support rods	• Sets of masses	• Stopwatch

Before the Lab

Read the sections in your text describing Newton’s Laws. You should learn draw a forces diagram and detemine the vector sum of all the forces on an object.

Theory

When two masses are suspended by a string over a pulley (Figure 3) each feels a downward force due to its weight (W=mg) and an upward force due to the tension (T) in the string (Figure 4). If these two forces are equal, then the net force on the mass is zero and, as Newton’s first law tells us, there will be *no acceleration* of the mass

If one mass is heavier than the other, then the masses may accelerate – one moving upward and the other down. Newton’s second law tells us that in this case there must be a *net force* on each mass:

$$\vec{F}_{net} = m\vec{a}$$

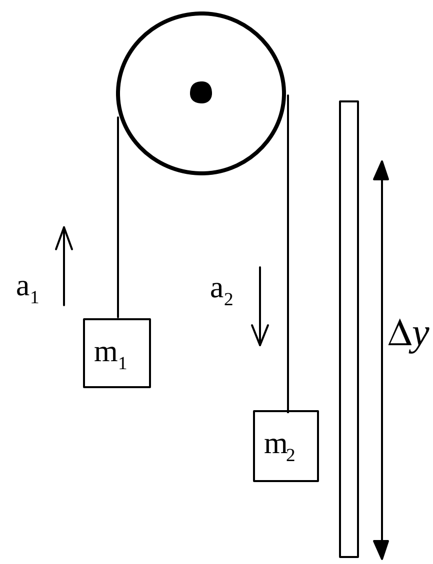


Figure 3: Mass and pulley system

If the string and pulley are considered to be nearly massless, then no force is need to either accelerate the string or cause the pulley to rotate. In this case the tension in the string is equal on both ends. The forces on the two masses are then:

$$F_1 = T - m_1 g = m_1 a \quad \text{and} \quad F_2 = T - m_2 g = -m_2 a$$

where we have used the fact that the acceleration of each mass is the same (since they are connected by the string) and assumed that mass 1 is accelerating upward and mass 2 is accelerating downward. Algebraically we can solve this to show that

$$m_2 g - m_1 g = (m_1 + m_2)a$$

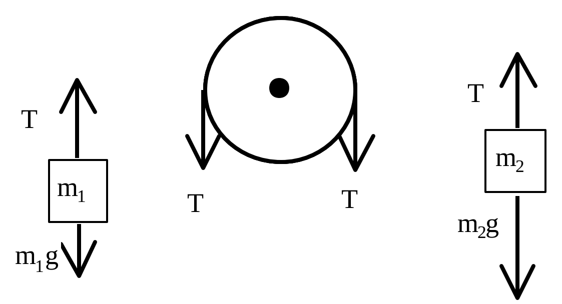


Figure 4: Force diagrams - draw each object separately

In effect the tension pulling in opposite directions cancels, so the net force becomes $F_{net} = T - m_1 g + m_2 g - T = m_2 g - m_1 g$ while the total mass being accelerated is $m_1 + m_2$. Thus $a = \dfrac{m_2 - m_1}{m_1 + m_2} g$ and if m_2 is greater than m_1 it will indeed accelerate downward. If m_1 is greater, the acceleration is negative meaning that each mass accelerates in the direction opposite to what was originally assumed.

Friction and Rotational Inertia

The real pulley and mass arrangement is not as simple as that described above. Some factors will affect the acceleration are 1) friction, which has not been included in the net force, 2) the mass of the string, and 3) the *rotational inertia* of the pulley. If we add the force of friction, *f*, to the net force, it becomes $F_{net} = m_2 g - m_1 g - f$. The fact the pulley is not massless means that it does require a net torque (force acting off-center) to make it rotate – this is supplied by the tension in the string. The *rotational inertia* of the pulley then adds an *equivalent mass* to the total mass being accelerated, so that $a = \dfrac{(m_2 - m_1)g - f}{m_1 + m_2 + m_{equiv}}$ where the equivalent mass for the pulley is approximately $m_{equiv} = \dfrac{1}{2} M_{pulley}$.

Determining the acceleration with a stopwatch and meter stick

The acceleration may be determined by measuring the time, t, required for the mass to fall from rest a specific distance Δy. If a is constant then we know that $\Delta y = v_0 t + \frac{1}{2} a t^2 = \frac{1}{2} a t^2$.

Procedure

You will be investigating the relationship between mass and acceleration by varying both m_1 and m_2 and observing a. We have shown from Newton's Second Law that

$$a = \frac{(m_2 - m_1)g - f}{m_1 + m_2 + m_{equiv}}.$$

Thus it seems worthwhile to try to systematically investigate the relationship between mass, force and acceleration by 1) keeping the net force constant by holding $(m_2 - m_1)g$ constant while varying $(m_2 + m_1)$ and 2) keeping the total mass, $(m_2 + m_1)$, constant while varying the net force by changing $(m_2 - m_1)g$

Vary the total mass (keep the net force constant)

Set up the apparatus as shown in the figure. Before assembling it record the weights of the mass hangers, the masses you will use, the pulley and the string. Use a piece of string long enough that you can observe a distance of at least 1 meter as the mass falls.

Begin with the two masses m_1 and m_2 each equal to 50 grams (this may be the mass of the hangers alone). When the two masses are equal the net force is zero. In the absence of friction, if the masses are set in motion they should continue to move at constant speed (zero acceleration). Hang the masses on the pulley and if necessary tap one side to set it in motion. Do you observe any significant friction? Record your observations.

Add 10 grams to m_2 but leave m_1 fixed. Start with m_2 near the pulley and measure the distance from the bottom of m_2 to the floor. This is Δy. Then release m_2 and record the time, t, which it takes to hit the ground. Make a few trial runs for practice and then record three independent measurements of t.

Add 100 grams to each side. This keeps the weight difference constant while increasing the total mass. Re-measure Δy (in case the string stretches) and record three independent measurements of time t.

Repeat twice more by adding additional increments of 100 grams to each side for a total of four trials with the 10 gram mass difference. Always re-measure Δy.

Now repeat for a total of four more trials using a 20 gram mass difference.

Vary the net force (keep the total mass constant)

Begin with a total of 260 grams for *each* mass. For the ascending mass, m_1 use a combination of (50 + 200 + 5 + 2 + 2 + 1) grams, for the descending mass, m_2, use (50 + 200 + 10) grams.

Transfer 1 gram from m_1 to m_2. Record the mass of each (259 grams and 261 grams) then measure Δy and make at least three good trials to record times.

Repeat by transferring additional mass from m_1 to m_2.

Transfer an *additional* 2 grams from m_1 to m_2.

Transfer an *additional* 2 grams from m_1 to m_2.

Transfer an *additional* 5 grams from m_1 to m_2.

Repeat starting with a total of 360 grams for *each mass*.

Analysis

For each trial, calculate the experimentally determined acceleration from the distance and time measurements. Show how the calculation is performed.

From the masses and mass differences calculate the theoretical acceleration. Include the effect of the equivalent mass of the pulley but neglect the effect of friction.

Calculate the percent difference between the theoretical and experimental values. (Percent difference is equal to $\left((Exp.{-}Theo.)/Theo.\right)\times 100\%$.)

For the data with constant total mass, the predicted relationship between acceleration and mass difference : $a = \dfrac{(m_2 - m_1)g - f}{m_1 + m_2 + m_{equiv}}$ can be represented on a graph of acceleration ($a_{Exp.}$) versus mass difference (m_1 - m_2). A graph of y versus x is a straight line if it fits the formula "$y(x)=mx+b$", where "m" is the slope and "b" is the intercept. Rewrite the equation for a in this form. What is the slope of the graph of a_{Exp} versus (m_2 - m_1)? What is the intercept. Answer these questions in your report.

Using a computer graphing program, make graphs of ($a_{Exp.}$) versus (m_2 - m_1) for each of your two data sets. Use a least squares fitting procedure to find the slope and intercept of your graph.

Determine the units of the slope and intercept determined from the graph.

From the slope and intercept, determine the gravitational acceleration, g and the force of friction, f. Make sure your units come out in m/sec^2 for acceleration and in Newtons for force.

Questions

Be sure that your report documents your calculations and explains how the value of g and the magnitude of the frictional force f can be determined from the graph.

How does the fact that the string has mass affect your experiment? Do you consider this an important effect?

Compare the magnitude of the force of friction to the other forces present in this experiment. Is friction a significant factor?

When the net force (due to mass difference) remains constant but the total mass increases, what happens to the acceleration?

When the total mass remains constant but the net force increases, what happens to the acceleration?

Explain specifically whether the answers to the previous two questions reflect Newton's Second Law.

Vary the total mass (keep the net force constant): 10 gram mass difference

Mass of pulley ____________________ m_{equiv} for pulley ________________________

Mass of string ____________________ Mass of hangers ________________________

Data

Ascending mass,	m_1							
Descending mass,	m_2							
Distance	Δy							
Time, t	1							
	2							
	3							
Time	Avg.							

Calculations

Calculated acceleration, a_{exp}							
Total mass $m_1 + m_2 + m_{equiv}$							
Mass Difference $(m_2 - m_1)$							
Net Force $(m_2 - m_1)g$							
Theoretical Acceleration, $a_{theo.}$							
% Diff between exp. and theo.							

Calculations (<u>show and explain work</u>)

Vary the total mass (keep the net force constant): 20 gram mass difference

Mass of pulley	____________	m_{equiv} for pulley	____________
Mass of string	____________	Mass of hangers	____________

Data

Ascending mass,	m_1							
Descending mass,	m_2							
Distance	Δy							
Time, t	1							
	2							
	3							
Time	Avg.							

Calculations

Calculated acceleration, a_{exp}							
Total mass $m_1 + m_2 + m_{equiv}$							
Mass Difference $(m_2 - m_1)$							
Net Force $(m_2 - m_1)g$							
Theoretical Acceleration, $a_{theo.}$							
% Diff between exp. and theo.							

Calculations (show and explain work)

Vary the net force (keep the total mass constant at 260 grams)

Mass of pulley	____________	m_{equiv} for pulley	____________
Mass of string	____________	Mass of hangers	____________

Data

Ascending mass,	m_1							
Descending mass,	m_2							
Distance	Δy							
Time, t	1							
	2							
	3							
Time	Avg.							

Calculations

Calculated acceleration, a_{exp}							
Total mass $m_1+m_2+m_{equiv}$							
Mass Difference (m_2-m_1)							
Net Force $(m_2-m_1)g$							
Theoretical Acceleration, $a_{theo.}$							
% Diff between exp. and theo.							

From graph:

Slope ____________ units g from graph ____________ units   [ ] theoretical g units

Intercept ____________ units f from graph ____________ units

Calculations (<u>show and explain work</u>)

Vary the net force (keep the total mass constant at 360 grams)

Data

Ascending mass,	m_1							
Descending mass,	m_2							
Distance	Δy							
Time, t	1							
	2							
	3							
Time	Avg.							

Calculations

Calculated acceleration, a_{exp}							
Total mass $m_1 + m_2 + m_{equiv}$							
Mass Difference $(m_2 - m_1)$							
Net Force $(m_2 - m_1)g$							
Theoretical Acceleration, $a_{theo.}$							
% Diff between exp. and theo.							

From graph:

Slope ____________________ units g from graph ____________________ units   [ ] theoretical g units

Intercept ____________________ units f from graph ____________________ units

Calculations (show work)

5. Friction: Investigation of a model for friction

Introduction

When two solid surfaces come in contact with each other there will be forces acting between the surfaces. These "contact forces" can be broken up into two components: a *Normal Force* which is perpendicular to the surface and prevents the surfaces from passing through each other and *Friction* which acts along the surface and prevents or hinders relative motion of the two surfaces. Both forces originate at the microscopic level in electromagnetic interactions between the atoms of the surfaces. Rather than attempting to understand all the details of this interaction, however, it is often useful to come up with a *model* for describing these forces. In this lab we will investigate one model for friction.

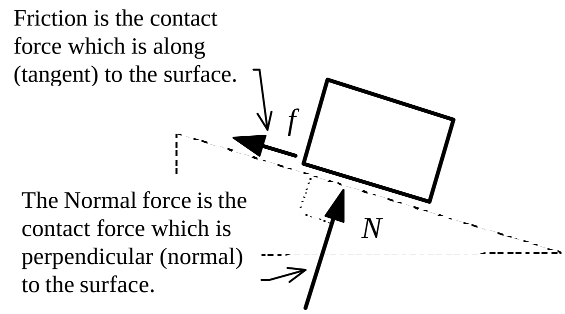


Figure 5: Forces on a block resting on an inclined surface. Contact forces are broken into two perpendicular components: friction and the normal force.

Equipment

- adjustable inclined plane with pulley
- ruler, protractor
- wooden blocks with hooks
- string and scissors
- graph paper
- masses, mass hanger
- balance to weigh masses
- blocks or books for propping

Before the Lab

Read the sections in your text describing friction. Read the theory section below and answer the pre-lab questions which follow.

Theory

Friction is the contact force between two surfaces which resists *relative* motion of the two surfaces: that is, it prevents or hinders the two surfaces from sliding relative to each other. The direction of the frictional forces is along, or tangent, to the surfaces and in a direction that will oppose the relative motion. Experimental observation has established that friction depends on the materials of the two surfaces (their composition and roughness) and upon the *Normal Force* between the two surfaces. The Normal force is the force of contact which is perpendicular to the surface and prevents the two surfaces from moving through each other. To a good approximation the frictional force appears to be independent of the area of contact between the surfaces.

Static Friction

When we attempt to make two surfaces slide relative to each other, for example by trying to push a book across the table, the force of friction will oppose the relative motion between the two surfaces. This means that as we increase the force with which we push the book, the force of friction will increase until it has reached a limiting value. While the two surfaces are not (yet) moving relative to each other, we call the force of friction *static friction*. To describe static friction mathematically, we can model it with the equation:

$$f_s \leq m_s N \qquad \text{(Equation 1: Static Friction)}$$

where f_s is the force of friction, m_s is called the *coefficient of static friction* and is a number that depends on the materials and roughness of the two surfaces and N is the normal force between the two surfaces. Static friction is less than or equal to the limiting value $m_s N$ because it starts at zero when there is no outside force trying to make the two surfaces slide relative to each other and then increases as the outside force increases until it reaches a limit and the two surfaces break free and begin to slide relative to each other.

Kinetic Friction

After the surfaces begin to slide relative to each other, friction holds a constant value and, while it can't prevent relative motion, it will hinder or slow down this motion. This type of friction is called *kinetic friction* and is represented by the equation:

$$f_k = m_k N \qquad \text{(Equation 2: Kinetic Friction)}$$

where f_k is the force of friction, m_k is the *coefficient of kinetic friction* and N is again the normal force. This equation shows that kinetic friction always has the same constant value any time the two surfaces are moving relative to each other (as long as the nature of the surfaces and the normal forces don't change). The coefficient of kinetic friction is often less than the coefficient of static friction, because once the two surfaces begin to slide relative to each other the frictional force may actually decrease slightly, due to factors such as a layer of air between the surfaces.

Note that these two equations only describe the *magnitudes* of the frictional forces. The *direction* is always along the surface (at 90^0 to the normal force, N) and directed so as to oppose the relative motion.

Experimental Method A: The inclined plane

Figure 6 shows that for a block on an inclined plane the component of weight along the plane, $mg \sin q$, will act to accelerate the block down the plane. Friction opposes this motion by acting in the direction up the plane. If the angle is small enough or the coefficient of friction is large enough, friction can prevent the motion of the block. Once the maximum limit for static friction is reached, however, the block begins to slide down the plane and, although the acceleration is decreased by the opposing force of kinetic friction, the block will accelerate as it slides down the plane.

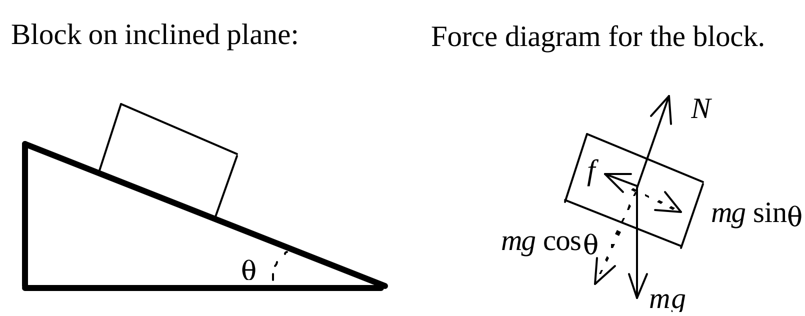


Figure 6: Block on an inclined plane

Consider an inclined plane, as shown in Figure 6, whose angle, θ, can be adjusted. If a block is placed on the plane while it is horizontal and the angle of the plane is slowly increased eventually the block will begin to slide. This is the point at which the limit of static friction has been reached. We can write the forces on the block in terms of components along (tangent to) the plane and components perpendicular (normal) to the plane.

Perpendicular to the plane the acceleration is always zero and the Newtons Laws tell us that

$$N - mg\cos\theta = ma_y = 0 \qquad \text{or} \qquad N = mg\cos\theta \tag{3}$$

Before the block slides, friction is less that or equal to the static limit and there is no acceleration along the plane either. Thus the forces along the plane are given by:

$$mg\sin\theta - f_s = ma_x = 0 \qquad \text{or} \qquad f_s = mg\sin\theta \qquad \text{(before block slides)} \tag{4}$$

In our model, $f_s \leq \mu_s N$ and therefore

$$f_s \leq \mu_s mg\cos\theta \qquad \text{(before block slides)} \tag{5}$$

The force of friction increases as the angle increases until the maximum value is reached. Exactly at this angle acceleration is still zero and:

$$f_s = mg\sin\theta_s = \mu_s mg\cos\theta_s \qquad \text{(just as block slides)} \tag{6}$$

From this we find that the angle at which the block slides, θ_s is related to the coefficient of friction by the equation:

$$\tan\theta_s = \sin\theta_s / \cos\theta_s = \mu_s \tag{7}$$

This provides an experimental way of determining the coefficient of static friction.

In a similar way the inclined plane can be used to determine the coefficient of kinetic friction. Once the block begins to slide, the frictional force in our model is determined by the equation $f_k = \mu_k N$. For the block on the inclined plane, this becomes

$$f_k = \mu_k mg\cos\theta \qquad \text{(while the block slides)} \tag{8}$$

If the coefficient of kinetic friction is less than the coefficient of static friction, then less force will be required to keep the block moving than was required to start the block moving. We can find an angle, θ_k, at which the block can move at constant speed (no acceleration). At this angle the kinetic friction must exactly balance the weight down the incline. We will need to "push" the block to start it moving but once it is started there will be zero net force down the incline and the block will move at constant speed. For this angle:

$$f_k = mg\sin\theta_k$$

and (while block slides at constant speed) (9)

$$f_k = \mu_k mg\cos\theta_k$$

and thus

$$\tan\theta_k = \mu_k \tag{10}$$

This equation may be used to find the coefficient of kinetic friction by finding an angle for which the block may need a "push" to get started but, once started, moves at constant speed.

Experimental Method B: The block and pulley.

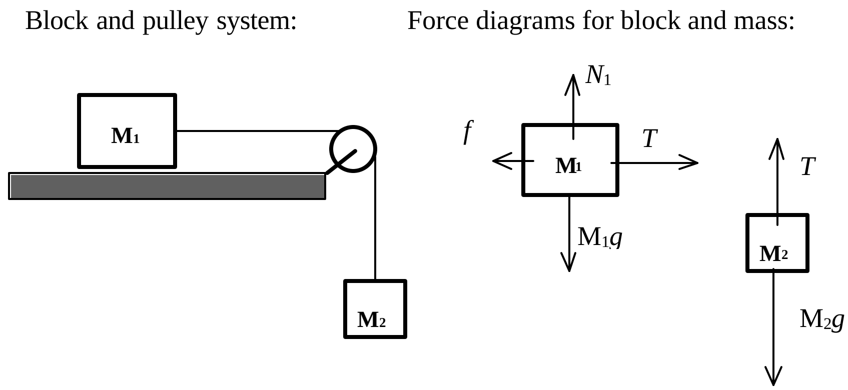


Figure 7: Block and pulley system

Another experimental arrangement for measuring the coefficients of friction is shown in Figure 7. Here the inclined plane is lowered to a horizontal position and force is applied to the block by a hanging mass connected to the block via a pulley. The force diagram shows the forces on both the block and the hanging mass. Because the Tension, T, transmitted by the string represents two objects acting on each other, they form an action-reaction pair and by Newton's third law, the tension, T, is the same for both objects. (This assumes that the string and pulley are both massless and require no force in order to move or rotate.) We again break forces into components and find that for the block, M_1, the forces normal to the horizontal plane and along the plane are:

$$N_1 - M_1 g = M_1 a_{1y} = 0 \qquad \text{and} \qquad T - f = M_1 a_{1x} \qquad (11)$$

and for the hanging mass, M_2 the forces are:

$$M_2 g - T = M_2 a_2 \qquad (12)$$

The acceleration of the block perpendicular to the plane, a_{1y}, is always zero. The acceleration of the block along the plane will be equal to the downward acceleration of the hanging mass and so $a_{1x}=a_2$.

If <u>there is no acceleration of the block and the mass (</u>$a_{1x}=a_2=0$) then $T=M_2g$ and at the angle for which the block just begins to slide (the limit for static friction):

$$f_s = T = M_2 g = \mu_s N_1 = \mu_s M_1 g \qquad \text{(just as the block begins to slide)} \qquad (13)$$

from which we determine that:

$$M_2 = \mu_s M_1 \qquad \text{(just as the block begins to slide)} \qquad (14)$$

This equation can be used to determine the coefficient of static friction by finding a mass, M_2, for which the block just begins to slide. Note that the basic relationship given by Equation (13) is a relationship between forces, M_2g and $\mu_s M_1 g$, but because the factor of g is common to both it cancels and the coefficient of friction is found in terms of a ratio of masses. A similar procedure can be used to find the coefficient of kinetic friction by finding a value for the hanging mass for which the block will slide at constant speed provided it is first given a "push" to overcome static friction. Again there is no acceleration so the forces exactly cancel and

$$f_k = T = M'_2 g = \mu_k N_1 = \mu_k M_1 g \qquad \text{(block moves at constant speed)} \qquad (15)$$

so that

$$M'_2 = \mu_k M_1 \qquad \text{(block moves at constant speed)} \qquad (16)$$

Pre-lab questions

Read the introduction to the laboratory and then answer the following questions before beginning the lab.

The direction of a frictional force is always in a direction which is ___________ to the relative motion or the tendency toward relative motion of two objects.

State what two kinds of coefficients of friction exist. Describe the conditions under which each kind is appropriate. State the relationship that is generally true that exists between their magnitudes.

Suppose a block of mass 25.0 kg rests on a horizontal plane, and the coefficient of static friction between the surfaces is 0.220. (a) What is the maximum possible static frictional force that could act on the block? ___________ Newtons. (b) What is the actual static frictional force that acts on the block if an external force of 25.0 Newtons acts horizontally on the block? ______________Newtons. Show your work

In the experiments to measure the coefficient of kinetic friction by sliding a block down an inclined plane, it is required that the block move a constant velocity. Why is this a requirement?

Frictional forces on a block on an inclined plane are always directed down the plane. (a) True (b) False. Explain.

A 5.0 kg block rests on a horizontal plane. A force of 10 N applied horizontally causes the block to move horizontally at constant velocity. What is the coefficient of kinetic friction between the block and the plane? (g= 9.8 m/s2) Show your work.

What are the units of the coefficients of friction?

If the equations described are valid, why doesn't the force of friction depend upon the apparent area of the two surfaces in contact?

How does the coefficient of kinetic friction depend upon the speed of a moving object?

Objectives of this lab

In this experiment the nature of the frictional forces between a smooth wooden block and a smooth flat board will be investigated to compare to what is expected using the equations given above (our model for friction). The board will be used in the horizontal position and as an inclined plane to accomplish the following objectives:

Determine μ_s and μ_k by two different methods.

Test whether $\mu_s > \mu_k$

Determine whether the coefficients of friction are independent of the normal force.

Part A: Board as an Inclined Plane

Procedure (A)

Measuring the coefficient of static friction

Place the block with its large surface down on the board. Adjust the angle of the board until the block just begins to slide on its own. Record the angle at which it slips as θ_s in the appropriate Data Table (Static Friction, A) as θ_s. Repeat the procedure at least three more times, allowing each lab partner to try adjusting the board. Each time place the block in a slightly different place on the plane in order to compensate for the effects of any irregularities in the surfaces. Note carefully which side of the block and which side of the board is used, and continue to use the same surfaces for all the measurements made in this experiment.

Measuring the coefficient of kinetic friction

Again place the block on the board, using the same surface as you used previously. This time you will incline the board until the block is able to move down the plane at constant speed after it is given a slight push to start it is motion. It may take several attempts to find an angle for which you can "nudge" the block to get it started but for which the block will then move at constant speed rather than accelerating. (The block may start and stall as it slides) Record the angle at which this occurs as θ_k. Repeat this process three more times for a total of four trials. Again try to use different parts of the board in order to average the effects of nonuniformity of the surface.

Calculations and Graphs (A)

1. For each θ_s calculate tan θ_s (see Equation 7) and record the values in the appropriate Data Table. Determine μ_s for the four measurements and then calculate the average value of μ_s determined by this method. From the variations in your measurements make an estimate of uncertainty in your value of μ_s.

For each θ_k measured calculate tan θ_k (see Equation 10) and record the values in the Data Table. Calculate and record the mean μ_k for the four measurements. Make an estimate of uncertainty.

Part B: Horizontal Board with Pulley

Procedure (B)

Measuring the coefficient of static friction

1. Determine the mass of the block using the balance. Record it in the Data Table (Static Friction, B) as M_1 in the space labeled "0 kg added".
2. Place the board in a horizontal position on the laboratory table with pulley beyond the edge of the table as shown in Figure 7. Place the block on the board using the same surface as you used previously.
3. Attach a piece of string to the hook in the block. Place it over the pulley and attach the mass holder to the other end of the string. **Adjust the position of the pulley until the string pulls parallel to the horizontal surface**. Carefully add mass to the mass holder to find the minimum mass needed to just cause the block to move. Record the value of mass determined as M_2 in the Data Table (Static Friction, B). Be sure to include the of the holder in the total for M_2. Repeat the procedure two more times for a total of three trials using different parts of the board each time.
4. Repeat procedure 3 but add 0.2 kg to the top of the block. Record the value of the mass of the block plus 0.2 kg as M_1. Again determine the minimum mass needed to just cause the mass M_1 to move. Do three trials and record each as M_2 in the Data Table (Static Friction, B).

Continue this process adding 0.4, 0.6, and finally 0.8 kg to the top of the block. In each case make measurements for three trials.

Measuring the coefficient of kinetic friction

Perform a similar set of measurements as just described, but this time determine the mass M_2 needed to keep the block moving a constant velocity after it has been started with a small push. Again take three trials for each case and use values of M_2 beginning with the mass of the block and incrementing in steps of 0.2 kg up to a total of 0.8 kg. Record the values of M_2 and M_1 for all the cases in the Data Table (Kinetic Friction, B).

Calculations and Graphs (B)

Calculate the mean (average) M_2 for the three trials which measured M_2 for each of the values of M_1 used in part B for both the static and kinetic friction cases. Record these values in the appropriate Data Table.

According to Equation (14) there is a linear relationship between M_2 and M_1, provided that m_s is a constant. Graph the static friction data for the horizontal plane case using M_2 as the ordinate (vertical or y-axis) and M_1 as the abscissas (horizontal or x-axis). If m_s is constant this graph should be a straight line. Show on the graph the straight line obtained from the linear least squares fit. From the slope of the line determine m_s.

Graph the kinetic friction data for the horizontal plane case using M_2 as the horizontal variable and M_1 as the vertical variable. Show on the graph the straight line obtained from the linear least squares fit. Determine μ_k from the slope of the line.

Questions

Discuss the agreement between the two different measured values of μ_s measured in part A and part B. Calculate the % difference for these two methods:

$$\%\ \text{difference of quantity "x"} \ = \ \frac{x_1 - x_2}{\text{average x}} \times\ 100\% \qquad (16)$$

Answer the same question for the kinetic friction data.

In part B, if the coefficient of friction depended on the mass M_1 then the equation $M_2 = m\, M_1$ would not be a straight line. Does it appear as if m is independent of M_1?

For which method (A or B) did you measure values of m_s and m_k for *different* values of the normal force. (Hint: examine equations (3) and (11) to find the normal force. Assume that any variations in angle for part A were due to experimental uncertainty.)

To what extent do your data confirm the expectation that the coefficients of friction, both static and kinetic, are independent of the normal force? State the evidence for your opinion.

Do your data confirm the expected relationship between μ_s and μ_k? State clearly what is expected and what your data indicate.

Data Table: Static Friction, A

Experimental Data			Calculations		
Surface used	θ_s		$\mu_s = \tan\theta_s$	Average μ_s	Estimated Uncertainty

Data Table: Kinetic Friction, A

Experimental Data			Calculations		
Surface used	θ_k		$\mu_k = \tan\theta_k$	Average μ_k	Estimated Uncertainty

Note: If you try different types of surfaces, you can record that under "Surface Used". For example "rough", "smooth", "large side", "small side".

Mass of hanger= ______________________

Data Table: Static Friction, B

Data			Calc.
Mass added	M_1	M_2	Average M_2 for this M_1
0 kg			

Data Table: Kinetic Friction, B

Data			Calc.
Mass added	M_1	M_2	Average M_2 for this M_1
0 kg			

What is the relationship between the Normal force and M_1? (See equation 11)

6. Centripetal Force: The center-seeking force.

Introduction

When an object is moving in a circle at constant speed the acceleration is *not* zero. Even though the speed may be constant the *velocity* is changing. Remember that the acceleration is the rate of change of velocity, not of speed, and velocity is a vector with both a magnitude and direction. Thus to move in a circle a force, **F**=m**a** is required. This force must point towards the center since when moving in a circle the direction of the velocity is continually bending towards the center. Using calculus it can be shown that for circular motion at constant speed the magnitude of the acceleration is $F=mv^2/R$, where m is the mass, v the speed and R the radius of the circle. In this lab we will attempt to verify this.

Equipment

- centripetal force apparatus
- masses, mass hanger
- stopwatch
- balance to weigh masses
- springs
- level
- ruler

Theory

In the absence of any external force, Newton's first law tells us that any object will either remain at rest or continue to move *in a straight line* at constant velocity. When moving in a circle, therefore, a force is required to change the *direction* of motion *even if the speed does not change*. This force must be directed towards the center of the circle. Whatever its source (gravity, a rope, a spring) we call this force the *centripetal force*, meaning center-seeking. We can see that this force is necessary for circular motion if we imaging what would happen if we removed it. When a mass tied to a rope is twirled in a circle, the inward force of the rope keeps it moving in a circle. If we cut the rope what would happen to the mass? Its inertia would keep it moving in a straight line at constant speed. Thus it would seem to "fly away" from the circle. In fact it is *not* traveling radially outward but rather in a line tangent to the circle. The natural condition of the mass is to travel in a straight line: we must apply a "centripetal force" by means of the tension in the rope to cause the direction of the velocity vector to change if we want it to travel in a circle.

Circular Motion:

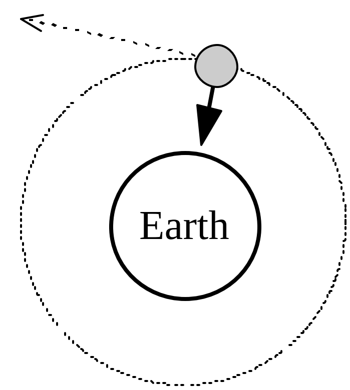


Figure 8: The moon travels (nearly) in a circle about the Earth due to the inward force of gravity. Without this force it would travel in a straight line

When traveling in a circle of radius R a distance of $2pR$ is covered every revolution. The period (time for one revolution) is denoted by the symbol T. The speed, v, is thus give by

$$v = \frac{\Delta" x"}{\Delta t} = \frac{2pR}{T} \tag{1}$$

For uniform circular motion at constant speed, it can be shown that the inwardly directed centripetal acceleration must be

$$a = \frac{v^2}{R} = \frac{1}{R}\frac{4p^2R^2}{T^2} = \frac{4p^2R}{T^2} \tag{2}$$

From Newton's Law, the net (total) force on the object must therefore be

$$F = ma = \frac{mv^2}{R} = \frac{m4p^2R}{T^2} \tag{3}$$

This force is created by real, physical forces (gravity, strings, springs etc) and is directed towards the center of the circle.

Procedure

The centripetal force apparatus is shown in Figure 9. The inward force is provided by a spring and the hanging Bob may be set in circular motion by twirling the center rod between your fingers. In this lab you will measure the force of the spring for different radii and then measure the period of circular motion required to keep the Bob moving in a circle of constant Radius. If the motion is too slow, the spring will pull the Bob inward towards the center. If the motion is too fast, the force of the spring will be insufficient to keep the Bob moving in a circle and it will travel tangent to the circle -- as its inertia tends to keep it moving in a straight line.

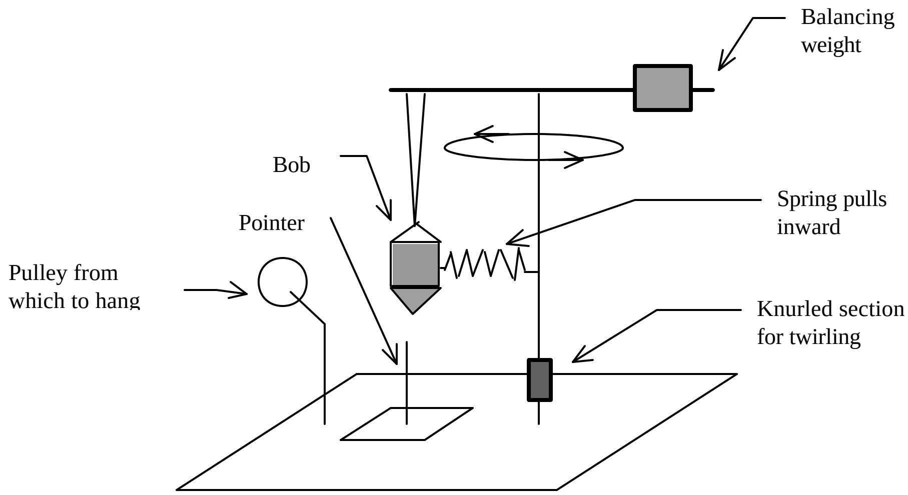


Figure 9: Centripetal Force Apparatus

Measuring the force of the spring for a given radius.

Weigh the Bob on the balance and record its mass as m_B.

Set the position of the Pointer to its smallest radius. Measure this radius (distance from the center rod) with a ruler. Record in Table One. Adjust the position of the Bob until it is hanging over the pointer when no spring is attached. Attach a spring. This will pull the Bob inwards. Attach a string to the Bob. Pass the string through the pulley and attach a mass hanger to the string. Add masses to the hanger until the Bob once again hangs straight over the pointer. Record this mass, m_H, in Table One and calculate the Weight of the hanging masses.

Draw two Force Diagrams Draw separate diagrams for the Bob and for the hanging mass.

The vertical forces on the Bob are the tension in the vertical string, T_V, and the weight of the Bob, W_B.

The horizontal forces on the Bob are applied by the spring, call this F_B, and the tension in the string that passes over the pulley to the hanging mass, call this T_H.

The hanging mass has only vertical forces, T_H and W_H. (T_H on the Bob and T_H on the hanging mass are equal and opposite due to Newton's third law.)

From your force diagrams show that in this situation where there is no acceleration $T_H=W_H$ and so the force of the spring $F_S=W_H=m_Hg$.

Measuring the period of circular motion

Remove the hanging masses and their string. The spring now pulls the Bob towards the center. Now you will twirl the Bob by rolling the knurled section of the center rod between your fingers until the Bob hangs vertically over the pointer. Notice that if you twirl slowly the Bob is pulled inward by the spring and if you twirl too fast the Bob travels outwards towards larger radius. At just the right speed the force of the spring will be equal to the centripetal force required to keep the Bob moving at constant speed. Practice twirling the rod until you can keep it moving at a constant speed with the Bob hanging directly over the pointer.

In order to determine the period of rotation you must keep the Bob twirling at constant speed as you count and time about some number, N, revolutions. A stopwatch will be used to measure the total time for N revolutions and period for one revolution can be calculated by dividing the total time by the number of revolutions (N). While the time and number of turns are being counted the rotation is maintained at constant speed by periodic twirling of the knurled section on the rod.

To measure the period have one lab partner (the twirler) turn the rod. Another lab partner (the timer) holds the stop watch while a third (the counter) counts the number of times the Bob passes over the pointer. When the twirler feels that they are maintaining a constant speed which keeps the Bob centered over the pointer they signal the timer and counter by saying "three, two, one, ***go***" After the timing starts the counter counts the number of times the Bob passes over the pointer. After counting 50 to 100 turns, the counter signals the end of the count, for example by saying "67, 68, 69, **stop**" and the timer stops the watch. Record the total number of revolutions, N, and the time, T_{total} in Table Two. The period measured is then $T = T_{total} /N$.

Repeat for an additional three trials, having lab partners switch roles (twirler, timer, counters). Record all trials and calculate an average period for this radius and this Bob mass.

Calculations and comparison

Calculate the velocity which corresponds to this period and radius.

Calculate the centripetal force required for this speed and this radius. Compare this to the force applied by the spring and calculate a percentage difference.

Repeat for different Bob masses

Loosen the knurled nut on top of the Bob and add a slotted mass (about 100 grams). Consider the Force Diagrams you drew previously. Is there any change in the force of the spring if the Bob's mass changes?

Measure the period of rotation required to keep the Bob moving at constant speed at this Radius as described above. Repeat this measurement three times.

Again compare the calculated centripetal force required for this mass to the force of the spring. How does the force change when the mass of the Bob changes? How has the velocity changed?

Repeat for a different radius.

Remove the added mass from the Bob. Move the pointer out to its largest radius. Move the Bob (with no spring attached) until it hangs vertically over the pointer. Now repeat the measurement of the force of the stretched spring for this larger radius by re-attaching the string via the pulley to the hanging masses. Add masses until the Bob once again hangs vertically. Record the masses and calculate the Weight of the hanging masses. Record this data in Table One.

Once again remove the hanging masses and measure the period of rotation needed to keep the Bob moving at constant speed and hanging vertically over the pointer. Record in Table Two.

Calculate the required centripetal force and compare to the measured force of the spring. How does the centripetal force change with radius?

Questions

1. When the Bob is moving in a circle, what provides the centripetal force? If it is suddenly removed, what would happen to the Bob?
2. What does Newton's First Law would predict would happen to the Bob if all force was removed? Is this consistent with your answer to Question 1?

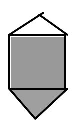
3. Examine equation (3). If everything in the equation were constant except that the mass increased, how would the centripetal force required to move in a circle change? If you double the mass what happens to the required centripetal force?
4. Examine equation (3). If everything in the equation were constant except that the speed increased, how would the centripetal force required to move in a circle change? If you double the speed what happens to the required centripetal force?
5. In your experiment, you changed the mass but kept the radius constant. Because the radius did not change, the force of the spring did not change. Instead, when you increased the mass, you had to change something else in order to make the required centripetal force stay the same. What did you change? Did you increase or decrease it? By how much (what ratio)? Is this what you would have predicted from equation (3)?
6. When you changed the radius but used the same mass, what parts of equation (3) changed? What did you have to change in order to keep equation (3) balanced? Did it change as you would have predicted from equation (3)?

Table One: Calibration of the Spring Force

Experimental Data			Calculations	
Mass of Bob	Radius (Length from center of Bob to center of Rod)	Hanging masses, m_H	Hanging Weight, m_Hg	Force of Spring for this Radius.

Force Diagrams for Bob and for the hanging mass. Draw and label the horizontal and vertical forces on each object. Show that if acceleration is zero the force applied by the spring is equal to the weight of the hanging mass.

Bob



Mass

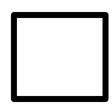


Table Two: Measurement of period, calculation of required centripetal force.

Bob Mass, m_B	Radius, R	Trial No.	N	Time T_{total}	Period T	Avg. T	$v = \frac{2pR}{T}$	$F = \frac{mv^2}{R}$	F_{spring} (from above)	% Diff.
		1								
		2								
		3								
		1								
		2								
		3								
		1								
		2								
		3								
		1								
		2								
		3								

6. Centripetal Force: The center-seeking force.

Chapter 2: Conservation Laws, Rotational Motion

7. The Ballistic Pendulum

Introduction

In this lab we will use conservation of energy and momentum to determine the velocity of a projectile fired into a pendulum and compare that to the velocity determined by looking at the trajectory of the projectile when it is launched across the room. In the Ballistic Pendulum (Figure 1) a spring loaded gun fires a projectile horizontally into the bob of a pendulum, where it becomes lodged. The pendulum swings backward to some maximum height where it is stopped. By measuring the maximum heights of the pendulum and projectile and using conservation of mechanical energy, we can determine the kinetic energy of the pendulum plus projectile immediately after the collision of the projectile and pendulum. Knowing the velocity of the pendulum/projectile immediately after they collide, we can then use conservation of momentum to calculate the velocity of the projectile alone before the collision. This velocity will then be compared to that determined by firing the projectile across the room and seeing where it lands.

Equipment

Ballistic Pendulum	Steel Ball	rubber bands	level, masking tape
metric rulers	paper and carbon	balance	2 meter sticks

Eye Protection Recommended when projectiles are flying around!

Before the lab

Review two dimensional projectile motion, conservation of energy and conservation of momentum. Pay particular attention to *when* energy or momentum are conserved. Mechanical energy (kinetic plus potential energy) is conserved when no work is done by non-conservative forces such as friction. *Internal forces can include forces of friction* which generate heat and cause loss of mechanical energy. Momentum is conserved when no *external* forces are acting. In a collision total momentum is conserved because the internal forces cause equal and opposite changes in the momentum of the colliding objects.

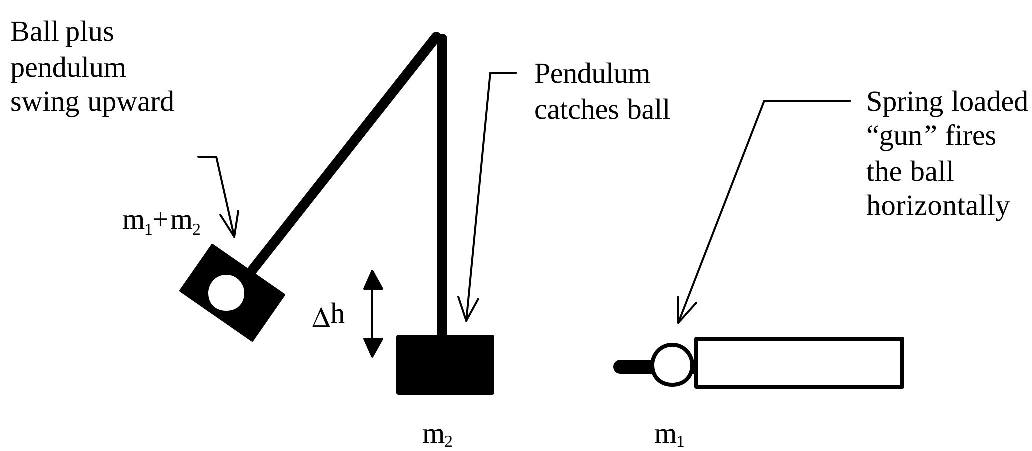


Figure 1: The Ballistic Pendulum. Momentum in conserved in the collision of projectile and pendulum. Mechanical Energy is conserved as the pendulum/projectile rises to its final height.

Theory

<u>Conservation of Momentum:</u>

One way to state Newton's Second Law: **F**=m**a** is in terms of the momentum, defined by:

$$\mathbf{p} = m\mathbf{v}$$

Since acceleration is the rate of change of velocity, it follows that if the mass is constant force must be the rate of change of momentum. When two objects collide the exert an equal and opposite force on each other. This means that their changes in momentum must be equal and opposite to each other. As long as there are no externally applied forces the momentum of the system must be a constant. This can be illustrated for an inelastic collision (one in which the two objects stick together after the collision as shown in Figure 2.) Suppose that before the collision one object, with mass m_1 is moving with velocity v_0. It strikes a second, initially stationary, mass, m_2, and after the collision they stick together and move with the same speed, $v_{(1+2)}$. . The momentum before the collision is given by $p_{initial}=m_1v_1$ and after the collision by $p_{final}=(m_1+m_2)v_{(1+2)}$ Since there are no external forces acting in the instant of the collision, conservation of momentum then determines that

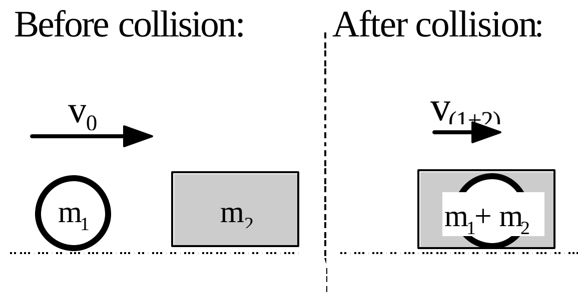


Figure 2: A totally inelastic collision

$$m_1v_0=(m_1+m_2)v_{(1+2)} \tag{1}$$

During this collision some of the energy may be dissipated as heat: kinetic energy is not conserved in the collision itself.

<u>Conservation of Energy:</u>

After the collision the pendulum/projectile combination (mass $m=m_1+m_2$) has initial speed $v=v_{(1+2)}$. This means it has kinetic energy $K= ½\, mv^2$. We can determine how much energy this is by seeing how high the pendulum swings: it stops when all the kinetic energy is converted to gravitational potential energy $U=mg\Delta h$. We assume friction is negligible and note that since the arm of the pendulum acts perpendicularly to its motion the work by the tension in the arm is zero. By measuring how high the pendulum/projectile swings we can determine what its velocity was in the instant *after* the collision. From this we can work backwards and use conservation of momentum (Equation 1) to determine the projectile's velocity *before* the collision.

<u>Projectile Motion:</u>

If we move the pendulum away from the projectile we can see how far it travels when we fire it across the room. The motion of the pendulum is determined by the initial velocity (in the x-direction) and the acceleration of gravity (in the y-direction). The equations for constant acceleration are then:

$$\Delta x = x-x_0 = v_{0x}t + ½\, a_x\, t^2 = v_0t$$

$$\Delta y = y-y_0 = v_{0y}t + ½\, a_y\, t^2 = -\, ½\, gt^2$$

From these equations and knowing the height Δy we can use a measurement of Δx to determine v_0 or *visa versa*.

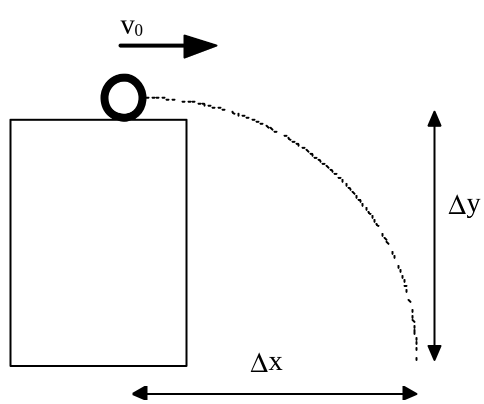


Figure 3: Projectile motion.

Procedure

Ballistic Pendulum

Determine the mass of the projectile, m_1 and the mass of the Ballistic Pendulum, m_2. Record these.

The mass of the Ballistic Pendulum is not all concentrated on the end. Find the *Center of Mass* of the Pendulum by finding the position where you can hold it horizontally and balance it. Record the distance from the bottom of the Bob to the Center of Mass and mark it with a pencil.

Replace the Pendulum. Place the projectile Ball on the gun.

Hold the ball release and push the ball back until the gun is cocked.

Fire the gun and record the number of notches up from the bottom at which the pendulum catches.

Repeat for a total of five trials. Calculate the average notch number. This will give you the final position of the pendulum.

Place the pendulum and ball so that they are hanging vertically (in the position they would be immediately after the collision). Carefully measure the height of the center of the ball and the height of the Center of Mass of the pendulum and record these in the data table. (Heights may be measured relative to either the base or the table).

Place the pendulum (with ball inside) at the notch corresponding to your average value for the position after the collision. Now carefully measure the height of the Ball and the height of the Center of Mass of the Pendulum. These are their final heights. Record them in your table.

Determine the pre-collision velocity

You will work backwards from the measured changes in height to find the pre-collision velocity, v_0.

Use Conservation of Energy to find the Kinetic Energy, and then the velocity, $v_{(1+2)}$ of the pendulum/ball combination right after collision (but before the initial height has changed.).

Calculate the change in potential energy of the ball due to its change in height.

Calculate the change in potential energy of the pendulum due to its change in height (by treating all the mass, m_2 as if its concentrated at the position of the Center of Mass of the pendulum.)

Calculate the total change in potential energy and from this find the initial kinetic energy of the pendulum/ball combination (Here initial means the instant after the collision but before the objects have begun to rise in height.)

Find the corresponding velocity $v_{(1+2)}$

Describe your calculations and record the results.

Now use Conservation of Momentum for the collision between the Ball and the Pendulum to find the pre-collision velocity of the Ball, v_0. (This assumes that although external forces like gravity are present the collision happens quickly enough that they have no affect). You have the common velocity, $v_{(1+2)}$ *after* the collision and know the masses. Describe your calculations and record results.

Shooting the Ball across the room.

You now have an pre-collision velocity for the ball with which it is launched by the gun, v_0. Measure the height of the gun off the floor and use v_0. as an initial velocity to find out where the ball would land on the floor if the Pendulum wasn't in the way. Describe your calculations and record your result.

Move the pendulum up the notched rack until it is out of the way. Making sure the space in front of your projectile is clear, measure out the distance to where you think the projectile will land. Tape a piece of paper to the floor and mark the spot with an X.

Fire your projectile and see if your prediction is accurate. Check your calculation and re-measure distances if you are off.

If you hit the paper, tape carbon paper on top of it and fire the projectile five more times to make marks where the Ball lands.

Carefully remove the carbon (put leave the paper) and record the positions, Δx, where the ball landed. (Δx is the horizontal distance from the initial ball position to where it lands.) Compare this to your predicted value.

Conclusions

Write a description of how you used the equation for Conservation of Energy, Conservation of Momentum and Projectile Motion in this lab (next page). What conditions made it valid to use these equations in these parts of the experiment?

Compare your prediction for the projectile range and the measured results. Discuss any differences and reasons for the differences. Are there systematic reasons you might expect the results to be larger or smaller than the prediction?

Is momentum conserved (constant) after the collision (as the pendulum swings upwards)? Recall that a) momentum is a vector $\mathbf{p}=m\mathbf{v}$ and has both *magnitude* and *direction* and b) total momentum is a constant if there are no external forces acting on the object.

In the collision itself Kinetic Energy is not conserved. Calculate the change in kinetic energy in the collision (from just before the ball hits to just after and before there is any change in height). Where might this energy have gone?

1. Conservation of Energy

Describe (in words) how you used Conservation of Energy in this lab. You must also explain *why* you were able to use conservation of energy for this part of the problem.

2. Conservation of Momentum

Describe (in words) how you used Conservation of Momentum in this lab. You must also explain *why* you were able to use conservation of momentum for this part of the problem.

3. Projectile Motion (Constant Acceleration)

Describe (in words) how you used the equations for constant acceleration in this lab. Show how you derived the result which you include in your Calculation Table.

Ballistic Pendulum Data

Mass of Ball		Notch Numbers for Five Trials	1) 2) 3) 4) 5)
Mass of Pendulum			
Location of Center of Mass of Pendulum		Average Notch Number	
Initial Height of Ball		Final Height of Ball	
Initial Height of Center of Mass of Pendulum		Final Height of Center of Mass of Pendulum	

Ballistic Pendulum Calculations:

	Formula or Equation used:	Result of Calculation:
Change in Potential Energy of Pendulum		
Change in Potential Energy of Ball		
Initial Kinetic Energy of Pendulum Plus Ball (right after collision)		
Velocity of Ball and Pendulum Right After Collision		
Momentum After Collision		
Momentum Before Collision		
Velocity of Ball Before Collision		

Predictions and Results for Projectile Motion (Ball Launched across room)

Measured Height from floor, Δy	Measured	
Initial Velocity, v_0	Determined above	
Calculation of Δx (This is your prediction)	Formula or Equation used:	Result:
Actual Δx measured	Five Trials: 1) 2) 3) 4) 5)	Average Δx Measured
Percent Difference: $\% = [(x_1 - x_2)/x_2] \times 100\%$		

8. Collisions on an air track

Introduction

We shall define a collision as an impact between two or more objects in a system or during a time in which *no external forces act*. The very fact that there are no external forces means that: **total momentum is always conserved in a collision**. In fact in three dimensions it is conserved separately in each direction since momentum is a vector. Although internal forces may cause equal and opposite changes in momentum, they do not necessarily cause equal and opposite changes in kinetic energy. Thus **kinetic energy is not necessarily conserved in a collision**. An *elastic* collision is one in which two objects bounce off of each other and kinetic energy is conserved. In an *inelastic* collision some of the total kinetic energy is lost from the system – it will be converted in to other forms of energy, for example heat generated by friction. A collision is referred to as *totally inelastic* when the two colliding objects stick together after the collision.

Equations

Momentum:

$$\vec{p} = m\vec{v} \qquad \vec{p}_{total} = \sum m_i \vec{v}_i$$

$$\%\Delta\vec{p} = \frac{\vec{p}_{final} - \vec{p}_{initial}}{\vec{p}_{initial}} \times 100\%$$

KineticEnergy:

$$K = \tfrac{1}{2} mv^2 \qquad K_{total} = \sum \tfrac{1}{2} m_i v_i^2$$

$$\%\Delta K = \frac{K_{final} - K_{initial}}{K_{initial}} \times 100\%$$

Equipment

Air Track	2 large carts	index cards for flags	magnets on carts
Air blower	1 small cart	2 photogates, mounts	ULI Timer, interface
Balance	Rods, clamps	Ruler, meter stick, tape	

Procedure

Carts gliding on an airtrack provide an opportunity to study collisions in one-dimension under nearly friction-free conditions. Begin by setting up the airtrack and photogates (about 45 cm apart). Attach pieces of index card to the carts to act as flags. Adjust the height of the photogates so the flags pass through them, as shown in Figure 4. The flags will block the infrared photodiodes in the photogates and send a timing signal to the computer. If you measure the width of the flags, Δx, and the photogate measures the time it takes for the flag to pass through the gates, Δt, then you will be able to measure the speed of the cart, $v = \Delta x/\Delta t$.

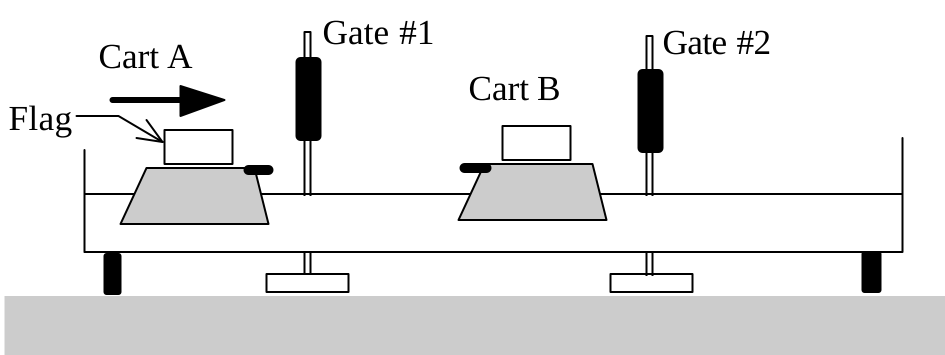


Figure 4: Air track with carts and photogates

Important: since the timing software accepts only one flag length, you must put flags of equal lengths on all carts**.** Record this length in the data table**.**

Set up the ULI Time software

Open the experiment file labeled "**Collision**" for this lab. This sets up the timers to measure two photogates independently and to calculate speeds from the measured times. Open the **Setup** window and enter the length of your flags under **Length**.

There are two columns for the times recorded by the two photogates. One column will record Gate #1 and the other column will record Gate#2. After you **Start** the timing successive times will appear in each column as each gate is blocked. Practice blocking and unblocking the gates with your hand until you understand how the times are recorded. **Set up the photogates so the time for Gate #1 is recorded in column 1 and the time for Gate #2 is recorded in column 2.**

Notes: for this lab you are only interested in collisions between the carts themselves, not in collisions of the carts with the end of the tracks. The experiment is "over" as soon as either cart bounces off the end of the track. You will keep recording times until you stop the software, but you will not be interested in any times due to rebounding carts passing through the photogates.

Level the airtrack.

It is very important that the airtrack be level and that the air supply properly hooked up.

You can check to see if the track is level by placing a cart on the track and seeing if it drifts to one end or the other. (The cart may drift slightly due to random fluctuations in the air supply but should not favor one side or the other).

Place a single cart on the track. **Start** the Timing as you give a small push so that it passes through both photogates. Ignore any additional times that are recorded after the cart rebounds from the end of the track. Since there are no external forces on the cart (after your initial push) its velocity should be constant. This means that the first two times recorded should be the same (within experimental uncertainty). If they are not then one of the following may be true:

If the cart slows down the air supply may be insufficient, resulting in friction. Check to see that the air hose is firmly connected.

If the cart slows down it may be going "uphill – you may need to raise the near end of the track.

If the cart speeds up it may be going "downhill" – you need to raise the far end of the track.

Important *These collisions do not work well if the initial velocities given to the carts are too large or too small. Always give the carts a fairly gentle initial push: a violent collision will cause the carts to tip and "dig in" to the air layer on which they float, causing excessive friction. On the other hand they should not be moving excruciatingly slowly either!*

Part A: Totally Inelastic collisions, only one car moving before the collision

Record the masses of both the large and small carts.

Totally Inelastic collision with carts of equal mass

Prediction: In this experiment Cart A with mass M collides with a second cart, Cart B, of nearly equal mass M. If Cart A is initially moving with velocity v_0 and Cart B is initially moving with zero velocity the momentum before the collision is $p_{initial} = M \cdot v_0 + M \cdot 0 = M \cdot v_0$. The Carts move together after the collision with velocity $v_{(A+B)}$ so the final momentum is $p_{final} = (M + M) \cdot v_{(A+B)} = 2M \cdot v_{(A+B)}$. If momentum is conserved in the collision then $p_{final} = p_{initial}$. Can you predict what will happen to the carts after the collision? Will they speed up? Slow down? What will be the ratio of $v_{(A+B)}$ to v_0?

Set up a collision between two gliders of nearly equal mass so that magnets attached to the carts will cause them to stick together after they collide. Place Cart A before the first Gate and Cart B stationary between the two gates, as in Figure 4. All carts should be initially clear of the photogates and both carts should be able to pass cleanly through the second gate before hitting the end of the track.

Give Cart A a gentle initial push. Observe qualitatively what happens to the carts after the collision (Do they speed up? Slow down? Come to a halt?) Record your observations and compare to your prediction.

Now set up the timing software. There will be three relevant times for this collision. Gate #1 will record the initial velocity of Cart A. The initial velocity of Cart B is zero, so no corresponding time will be recorded. Both carts pass through Gate #2 after the collision, so the final velocities of each cart will be recorded there. Because the carts are stuck together after the collision, if both carts have flags of equal length on them, there really is no friction, and the track is level, these times should be equal. If they are not re-check the leveling and air supply for your track. The two times can be significantly different only if the two carts are speeding up or slowing down as they travel together after the collision.

Make at least two trials for this type of collision and record the relevant times and velocities. For each trial, copy the computer output directly onto the first table. Then determine which entries match which carts and record the appropriate data and calculations in the second table.

Compare your results to your qualitative observations and prediction. Do they make sense?

Determine whether momentum is conserved in this collision by calculating the percentage change in total momentum for each trial. *Note: you do not average the different trials since you cannot control the initial velocities.*

Determine what happens to kinetic energy in this type of collision in each trial by calculating the percentage change in kinetic energy from before to after the collision

Totally Inelastic collision with carts of unequal mass

Repeat using carts of different masses, placing the heavier cart in front and initially at rest. *Both carts must have flags of equal length to use the program's velocities.* Although there is only one final velocity once the carts are stuck together, by using two flags, you can again determine whether they are speeding up or slowing down inappropriately.

First make a prediction and qualitative observations for this type of collision.

Again make at least two trials, compare results to your qualitative observations, and calculate percentage changes in total momentum and kinetic energy for each trial.

Part B: "Elastic" collisions, only one car moving before the collision.

"Elastic" collision with carts of equal mass

Set up an elastic collision with two carts of nearly equal mass. Place Cart A before the first photogate and Cart B should be stationary between the photogates, as shown in Figure 4. Again all carts should be clear of the photogates and there should be room for the carts to pass completely through the gates before colliding with the ends of the track.

Qualitative observations: Give Cart A a gentle initial push and observe what happens to each cart after the collision. Record you observations.

Prediction: In this experiment Cart A with mass M and initial velocity v_0 has collided with Cart B of nearly equal mass M and zero initial velocity. Before the collision the momentum is $p_{initial} = M \cdot v_0 + M \cdot 0 = M \cdot v_0$. After the collision the momentum is $p_{final} = M \cdot v_A + M \cdot v_B$. The final velocities are positive if in the same direction as v_o and negative if in the opposite direction to v_0. If momentum is conserved then $p_{final} = p_{initial}$. What is your qualitative observation of what happened to v_A after the collision? Can you predict what the value of v_B should be?

Measure the velocities: There are up to three relevant times for this type of collision. Cart A initially passes through Gate #1 so the time corresponding to its initial velocity is in column 1. Cart B is initially stationary so its initial velocity is not recorded. After the collision Cart B passes through Gate #2 and its time is recorded in column 2. Cart A may a) rebound and pass through Gate #1 again, b) stop, or c) follow Cart B through Gate #2 in which case its final velocity is recorded in column 2. Figure out what times are recorded in which columns and then make at least two trials to record the velocities before and after the collision. For any initial or final velocities which are zero, no time will be recorded and you should just record your observation that the velocity is zero.

Compare your results to your predictions and qualitative observations.

For each of the trials, calculate the percent change in total momentum before and after the collision. Is momentum conserved (within experimental uncertainty)?

Calculate the percent change in kinetic energy for this type of collision. The smaller the percentage change in kinetic energy, the closer this collision is to being perfectly elastic.

"Elastic" collision with carts of unequal mass

Repeat the above procedure for carts with unequal mass, placing the heavier cart in front.

Part C: Collisions with both carts initially moving.

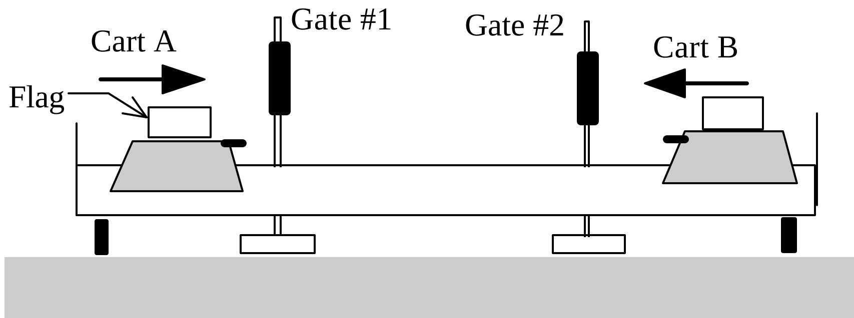


Figure 5: Collision with both carts initially moving

Head-on collisions

Set up the carts as in Figure 5. Push the carts toward each other at approximately the same time. Cart A initially passes through Gate #1 and Cart B initially through Gate #2. What happens after the collision depends on the masses of the cart and their initial velocity. If the collision is elastic and they both rebound from each other the Cart A's final velocity is recorded in Gate #1 and Cart B's final velocity in Gate #2. They may also both travel to either the left or the right or one or both carts may stop (or move so slowly that the velocity is not recorded).

Make a trial run for this type of collision to figure out what columns will record which velocities and then determine the "before" and "after" speeds for each cart. Calculate the initial and final momenta. *Remember that velocities are positive if in the same direction as Cart A's initial velocity and negative if in the same direction as Cart B's initial velocity*. This means that even though all times recorded are positive, Cart B has a negative initial velocity and either or both carts may have a negative final velocity.

With two carts of equal mass set up a collision which has zero momentum both before and after the collision. (No fair setting up the case where they are not moving before the collision!)

Questions

How do your results compare to both your predictions and your qualitative observations. Discuss possible sources or error or uncertainty.

Do the results support Conservation of Momentum? What possible sources of error could explain any discrepancies?

In first few collisions one cart was initially at rest and then it was moving after the collision. What caused the acceleration of this cart? What effect did this have on the other cart?

Which type of collision ("elastic" or totally inelastic) causes a greater change in kinetic energy?

Part A No. 1: Totally Inelastic collision with carts of equal mass

Length of flag:	Masses of large carts:	Mass of small cart:
<u>Prediction:</u>		
<u>Qualitative Observations:</u>		

<u>Trial 1</u> Data Table as copied directly from computer (*cross out any irrelevant times*):

	Gate #1		Gate #2	
Row	**Δ t1, sec**	**Velocity 1, m/sec**	**Δ t2, sec**	**Velocity 2, m/sec**
1				
2				
3				

<u>Trial 1</u> Your Data and calculations for this collision. Show your work.

	Before the collision	**After the collision**	**% Diff.**
Mass A=	v_A=	v_A’=	
Mass B=	v_B=	v_B’=	
	$p_{initial}$=	p_{final}=	% Δp=
	$K_{initial}$=	K_{final}=	% ΔK=

<u>Trial 2</u> Data Table as copied directly from from computer (*cross out any irrelevant times*):

	Gate #1		Gate #2	
Row	**Δ t1, sec**	**Velocity 1, m/sec**	**Δ t2, sec**	**Velocity 2, m/sec**
1				
2				
3				

<u>Trial 2</u> Your Data and calculations for this collision. Show your work.

	Before the collision	**After the collision**	**% Diff.**
Mass A=	vA=	vA’=	
Mass B=	vB=	vB’=	
	pinitial=	pfinal=	% ? p=
	Kinitial=	Kfinal=	% ? K=

Part A No. 2: Totally Inelastic collision with carts of unequal masses

Length of flag:	Masses of large carts:	Mass of small cart:
Prediction:		
Qualitative Observations:		

Trial 1 Data Table as copied directly from computer (*cross out any irrelevant times*):

	Gate #1		Gate #2	
Row	**Δ t1, sec**	**Velocity 1, m/sec**	**Δ t2, sec**	**Velocity 2, m/sec**
1				
2				
3				

Trial 1 Your Data and calculations for this collision. Show your work.

	Before the collision	**After the collision**	**% Diff.**
Mass A=	v_A=	v_A'=	
Mass B=	v_B=	v_B'=	
	$p_{initial}$=	p_{final}=	% Δp=
	$K_{initial}$=	K_{final}=	% ΔK=

Trial 2 Data Table as copied directly from from computer (*cross out any irrelevant times*):

	Gate #1		Gate #2	
Row	**Δ t1, sec**	**Velocity 1, m/sec**	**Δ t2, sec**	**Velocity 2, m/sec**
1				
2				
3				

Trial 2 Your Data and calculations for this collision. Show your work.

	Before the collision	**After the collision**	**% Diff.**
Mass A=	vA=	vA'=	
Mass B=	vB=	vB'=	
	pinitial=	pfinal=	% ? p=
	Kinitial=	Kfinal=	% ? K=

Part B No. 1: "Elastic" collision with carts of equal mass

Length of flag:	Masses of large carts:	Mass of small cart:
Prediction:		
Qualitative Observations:		

Trial 1 Data Table as copied directly from computer (*cross out any irrelevant times*):

	Gate #1		Gate #2	
Row	**Δ t1, sec**	**Velocity 1, m/sec**	**Δ t2, sec**	**Velocity 2, m/sec**
1				
2				
3				

Trial 1 Your Data and calculations for this collision. Show your work.

	Before the collision	**After the collision**	**% Diff.**
Mass A=	v_A=	v_A'=	
Mass B=	v_B=	v_B'=	
	$p_{initial}$=	p_{final}=	% Δp=
	$K_{initial}$=	K_{final}=	% ΔK=

Trial 2 Data Table as copied directly from from computer (*cross out any irrelevant times*):

	Gate #1		Gate #2	
Row	**Δ t1, sec**	**Velocity 1, m/sec**	**Δ t2, sec**	**Velocity 2, m/sec**
1				
2				
3				

Trial 2 Your Data and calculations for this collision. Show your work.

	Before the collision	**After the collision**	**% Diff.**
Mass A=	vA=	vA'=	
Mass B=	vB=	vB'=	
	pinitial=	pfinal=	% ? p=
	Kinitial=	Kfinal=	% ? K=

Part B No. 2: "Elastic" collision with carts of unequal masses

Length of flag:	Masses of large carts:	Mass of small cart:
Prediction:		
Qualitative Observations:		

Trial 1 Data Table as copied directly from computer (*cross out any irrelevant times*):

	Gate #1		Gate #2	
Row	**Δ t1, sec**	**Velocity 1, m/sec**	**Δ t2, sec**	**Velocity 2, m/sec**
1				
2				
3				

Trial 1 Your Data and calculations for this collision. Show your work.

	Before the collision	**After the collision**	**% Diff.**
Mass A=	v_A=	v_A'=	
Mass B=	v_B=	v_B'=	
	$p_{initial}$=	p_{final}=	% Δp=
	$K_{initial}$=	K_{final}=	% ΔK=

Trial 2 Data Table as copied directly from from computer (*cross out any irrelevant times*):

	Gate #1		Gate #2	
Row	**Δ t1, sec**	**Velocity 1, m/sec**	**Δ t2, sec**	**Velocity 2, m/sec**
1				
2				
3				

Trial 2 Your Data and calculations for this collision. Show your work.

	Before the collision	**After the collision**	**% Diff.**
Mass A=	vA=	vA'=	
Mass B=	vB=	vB'=	
	pinitial=	pfinal=	% ? p=
	Kinitial=	Kfinal=	% ? K=

Part C: Head-on collisions

Length of flag:	Masses of large carts:	Mass of small cart:
Prediction:		
Qualitative Observations:		

Trial 1 Data Table as copied directly from computer (*cross out any irrelevant times*):

	Gate #1		Gate #2	
Row	**Δ t1, sec**	**Velocity 1, m/sec**	**Δ t2, sec**	**Velocity 2, m/sec**
1				
2				
3				

Trial 1 Your Data and calculations for this collision. Show your work.

	Before the collision	**After the collision**	**% Diff.**
Mass A=	v_A=	v_A’=	
Mass B=	v_B=	v_B’=	
	$p_{initial}$=	p_{final}=	% Δp=
	$K_{initial}$=	K_{final}=	% ΔK=

Trial 2 Data Table as copied directly from from computer (*cross out any irrelevant times*):

	Gate #1		Gate #2	
Row	**Δ t1, sec**	**Velocity 1, m/sec**	**Δ t2, sec**	**Velocity 2, m/sec**
1				
2				
3				

Trial 2 Your Data and calculations for this collision. Show your work.

	Before the collision	**After the collision**	**% Diff.**
Mass A=	vA=	vA’=	
Mass B=	vB=	vB’=	
	pinitial=	pfinal=	% ? p=
	Kinitial=	Kfinal=	% ? K=

9. Conservation of Energy in a Pendulum

(See next page for equipment list)

Introduction

The Work-Energy theorem tells us that changes in *Kinetic Energy*,

$$K = \tfrac{1}{2}mv^2$$

are caused by *Work* done by forces. A special case pertains when the Work is done by a *Conservative* force. A conservative force is one for which the work done when traveling between any two points is independent of the path taken between the initial and final point. An example of this is the force of gravity (weight): work by gravity is always equal to *-mg* times the change in height, *Dh*. The amount of work done does not depend on how the object in questions gets from one height to another, only on the final change in height. Furthermore, if the object returns to its initial height all of the energy lost to work against gravity can be recovered.

We can think of the negative work done against gravity as a *Potential Energy* that is being stored up and which can be recovered and converted back to Kinetic Energy by allowing gravity to do positive work. We define the change in Potential Energy, *U*, as

$$U = -\text{ Work by a conservative force}$$

If there are no non-conservative forces (such as friction) the total *Mechanical Energy, E,* defined as

$$E = K + U$$

is a constant. We call this *Conservation of Energy.*

For the gravitational force, the potential energy is given by:

$$U_{gravity} = mg\, Dh$$

where *mg* is the weight (force due to gravity) and *Dh* is the change in height. Note that Potential Energy can be negative: we define it as zero at some reference point, for example when we define the floor as zero height. Below this point the Potential Energy is negative, above this point the Potential energy is positive.

In this lab we will investigate conservation of energy for a swinging pendulum. The experimental arrangement is shown below. A motion sensor is used to record the position of the Bob and calculate velocity. From the recorded position and velocity you will use a spreadsheet to calculate Kinetic and Potential Energy: $K = \frac{1}{2}mv^2$ and $U = mg\, Dh$

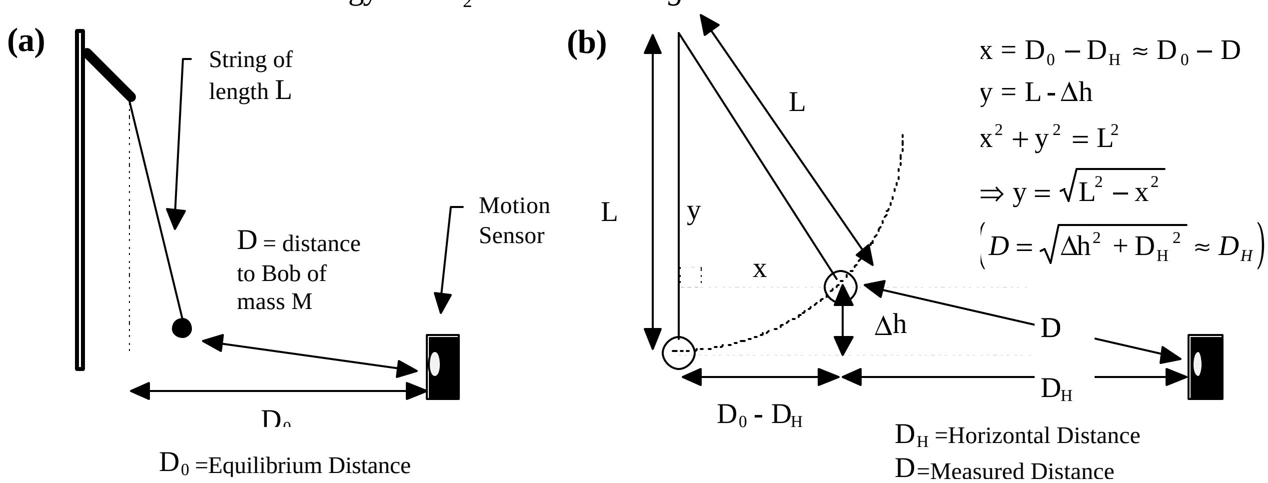


Figure 6: Experimental Set-up for recording the motion of a pendulum

Equipment

Pendulum clamp, string, Bob
Motion Detector
ULI Interface
Meter Stick
Spreadsheet (Excel)
Balance

Procedure

Aligning the pendulum and motion detector

You have a pendulum and a motion detector. Measure and record the mass of the pendulum bob, m. Hang the pendulum so that when the Bob is hanging straight down its height is in the lower part of the motion detector. (Place the clamp above eye level so you don't walk into it!) Measure and record the length of the string. Move the detector away from the Bob to perform the next observations.

Predictions: Observe the pendulum and place your bets!

Pull the pendulum away from the hanging position by 10-20 cm and release it. Describe below how the velocity and the height of the Bob change as it moves.

2. Where is Potential Energy (PE) a maximum? Where is it a minimum?

Where is Kinetic Energy (KE) a maximum? Where is it a minimum?

What do you think the sum of KE and PE should be as the pendulum moves? Does this match your observations. If not, take another look.

Set up the motion detector

Place the motion detector in front of the pendulum so as to align its sensor with the Bob and then move the detector until it is far enough away to be able to sense the motion of the Bob. You can use a meter stick aligned on the table to help you move it back while keeping the detector pointed at the Bob.

Start the MacMotion program and display two graphs: both position and velocity. Test your alignment until you can generate good graphs of position and velocity with no flat lines indicating that the Bob has neither gone out of range nor is hitting a nearer obstacle. If your graph is too small you can click on the top number on the vertical axis to change the scale.

Record the motion of the pendulum

Carefully making sure the motion sensor does not move while you are doing your experiments, pull the pendulum Bob back a distance of 5 cm or less and release it. SMALL AMPLITUDE GIVES BETTER RESULTS. The detector measures the distance D (see Figure 6) but for our calculations it is easier to use the distance D_H. FOR SMALL AMPLITUDES $D \approx D_H$.

As it is released start the motion detector program and record position as a function of time. With your program set to display two graphs you should simultaneously see velocity as a function of time.

Find the equilibrium distance

With your meter stick you have measured the equilibrium distance, D_0, which is the distance from the detector to the pendulum when it is hanging straight down. Now examine your distance graph and identify the points that record the Bob being farthest from the detector, closest to the detector and at the equilibrium distance, D_0, from the detector. Use the **Analyze Data A** feature of the program to find the distance that corresponds to the middle distance, D_0. Make sure that this number is consistent with what you can measure with the meter stick and record it.

Using Figure 6 (b) determine how to calculate the change in height of the pendulum, $\Delta h = L - y$, by measuring the distance from equilibrium, $D \approx D_H$. That is, derive a formula for Δh=____, where everything on the right side of the equal sign is known (e.g., L, D_0) except for D <u>Draw a diagram and show how you derive your formula.</u>

Save your data!

Look at the columns of data (From the Windows menu find Data Table A) and identify which columns record time, distance, and velocity.

From the **File** menu use **Export Data A** to save your data (CHECK THE BOX AT THE BOTTOM TO SAVE AS TEXT on the desktop or on a floppy).

Using the spreadsheet to calculate KE and PE and make a bar chart.

For the following students should save data on enough disks so that they may work in groups of two to generate the spreadsheet calculation and graph. This way everyone will get experience with the spreadsheet.

Open the Excel program and then open the data file you have just saved. If you have saved it as text the Excel program should be able to automatically import all the data in columns. Identify the columns which record time, position and velocity. Use the Edit menu to delete the columns recording acceleration and force.

Now you will create two more columns to record Kinetic and Potential Energy. Determine a formula that will allow you to calculate Kinetic Energy in Joules from velocity, v, as measured by the detector. Determine an Excel formula that will allow you to calculate Potential Energy in Joules from distance, D, as measured by the detector. Enter these in the spreadsheet and record these formulae. You will be given help in writing the formula in the first row of data and then filling out the rest of the sheet.

Your spreadsheet now contains columns showing KE and PE. Use the chart wizard to generate a column chart with stacked columns showing both KE and PE as function of time. Print your graph. (After clicking on the chart wizard button you need to select a place on the page for the chart area).

Add a legend to the graph explaining what it is. Print your graph.

On your graph or another piece of paper <u>write a verbal description and explanation of what is shown in the graph.</u>

<u>Discussion of Results:</u>

Discuss your charts of KE and PE by comparing them to the predictions you made previously. When KE is a maximum, what is PE? When PE is a maximum, what is KE?

How does your graph show total Energy, E? What happens to the total Energy?

Is there any evidence that the approximation $D \approx D_H$ is causing you trouble?

For your report write a one page cover sheet with an explanation of the theory of the experiment in your own words. Include data below. Print out chart from Excel showing KE, PE and total E with explanation and discussion of the graph. Be sure and discuss how the graph relates to motion of the pendulum. Answer questions listed in the handout.

Parameters needed for calculations:

Mass of Pendulum Bob, m	
Length of Pendulum, L	
Equilibrium Distance, D_0, (from motion sensor graph):	
<u>Formula</u> for converting D_0-D to change in height Δh (gives Δh in terms of L, D_0, and D)	Δh =
<u>Formula</u> for spreadsheet to calculate Kinetic Energy in Joules from velocity, v. (check the units!)	KE =
<u>Formula</u> for spreadsheet to calculate the Potential Energy in Joules from distance D. (check the units!)	PE =
Excel Formula for KE as entered in spreadsheet	=
Excel Formula for PE as entered in spreadsheet	=

Notes for Formulae in Excel:

Formulae start with an =

Reference cells by clicking on them as you enter the formula (have someone show you the first time).

Multiply numbers with a *

Raise numbers to a power with a ^ (for example x^2 is written as x^2)

Use plenty of parenthesis to factor.

Functions can be found by using the button with the script f_x.

A Square Root is given by SQRT(x) or by (x)^0.5.

Enter the formula only in the first cell of the column. After that select the first cell, move the mouse to the corner until you get a black crosshair and then drag down to fill the rest of the column or select an area to fill and use FILL DOWN from the Edit menu.

10. Balancing Torques and the Center of Gravity

Introduction

Just as force is required to accelerate a mass, a *torque* is required to produce angular acceleration. A torque is a force applied at a distance offset from some axis of rotation. This distance is often called the *lever arm* and the larger the lever arm is the more rotational acceleration can be produced by the force. The torque, τ, is defined by

$$t = r_\perp \times F$$

where $r_\perp$ is the *lever arm:* the distance from the axis of rotation to the line of the applied force, as shown in Figure 7. For rotation in two dimensions, torques can cause counter-clockwise (positive) rotation or clockwise (negative) rotation.

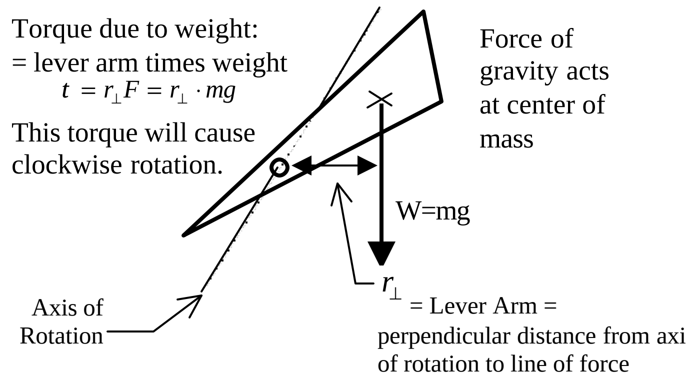


Figure 7: The weight of an object can cause a torque about an off-center axis

Equipment

Meter Stick	Pivot	3 Mass Hangers, 3 meter stick clamps
3 known weights	1 unknown lead weight	Triple Beam Balance

Theory

In this lab you will study the special case of an object in *static equilibrium*: it is neither accelerating nor rotating. This means that there is both no net force and no net torque on the object. Any forces and torques applied to the object must be balanced. For example any torque which would cause clockwise rotation must be balanced by a torque which would cause counter-clockwise rotation.

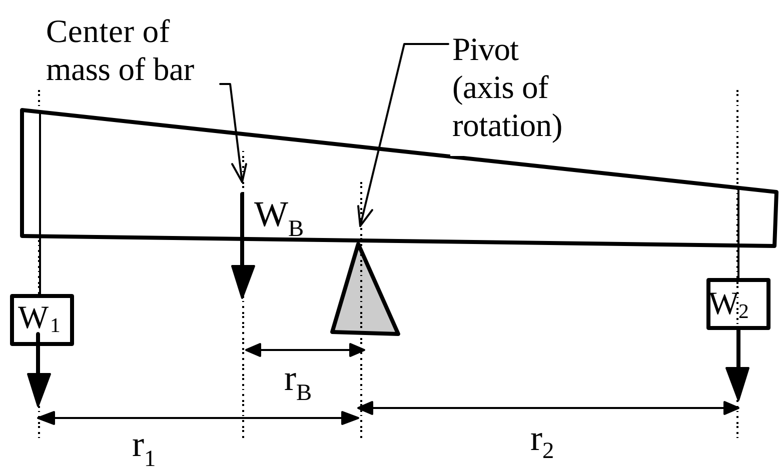


Figure 8: Objects balanced on a pivot: there is no net force and also no net torque. The weight acts at the center of mass.

One of the external forces on the object might be its weight. Although the force of gravity acts on every point throughout the object, the net effect is the same as if all the weight were concentrated at the *center of mass*. We say that the *weight acts at the center of mass*, which is sometimes also called the *center of gravity*.

Figure 8 shows a bar balanced on a pivot. If the bar were to rotate then the pivot would be the axis of rotation. In order for it not to rotate the torques produced by all the forces must balance. The weight, $W_B = m_B g$, of the bar itself acts at the center of mass of the bar. The lever arm for this force is shown as r_B so the torque it would produce is $\tau = r_B W_B$ which would tend to make the bar rotate counter clockwise. Similarly the hanging mass M_1 also produces a positive counter-clockwise torque , $r_1 W_1$, while the second hanging mass, M_2, produces a negative, clockwise torque, $- r_2 W_2$. For the bar to be balanced the net torque must be zero:

$$\tau_{total} = r_B W_B + r_1 W_1 - r_2 W_2 = 0$$

or

$$\tau_{CCW} = \tau_{CW} \qquad \text{so that} \qquad r_B W_B + r_1 W_1 = r_2 W_2$$

Procedure

A meter stick balanced on a pivot will be used to investigate how balancing torques creates an equilibrium.

A. Meter stick supported at center of gravity:

Determine the weight of your meter stick and the three clamps you will use (one for the center and one on each end). Record this data.

Adjust the position of the meter stick in the center clamp until you can balance it on the pivot. Balance the meter stick on the pivot by moving it inside the center clamp until you reach an equilibrium position. The center of mass should be over the pivot now. Record this position.

Two known weights (Do not use equal weights!)

Select two unequal weights and attach them to either side of the stick by strings or by attaching them to clamps, in which case the weight of the clamp is included in the total weight. Adjust their positions until you find a balance. Record the weights used, (W), and their positions as read on the meter stick (X). Make a sketch of the arrangement and label the positions of the weights and the center of mass.

Calculate the lever arm for each weight. Indicate this distance on your sketch as well.

Calculate the net clockwise torque and the net counterclockwise torque. Find the percent difference between the two. How does this compare to the condition required for balance?

As you move the two weights how is the rotational equilibrium disturbed? Is there more than one way to position the two weights to achieve balance? Vary the positions of the weights until you have determined a general condition for balancing the weights. Record your observations and conclusions.

In the second data table record the weights and positions corresponding to your new equilibrium. Calculate the CCW and CW torques and their percent difference.

Three known weights (not all the same)

Repeat using three weights (at least two different) at three different positions. Find an equilibrium. Sketch your configuration and record your weights and positions. Compute clockwise and counter-clockwise torques and their percent difference.

Now choose two different positions for the first two weights. Calculate the lever arm required for the third weight to balance these two. Record your prediction then experimentally find the actual position required. Calculate the percent difference between the predicted and measured positions.

Unknown Mass:

Choose a lead weight to use as an unknown mass. With the meter stick balanced at its center of mass, hang the unknown mass on one side. On the other side place a known mass and adjust its position until you find a balance. Describe the calculations needed to find the unknown mass and record your results. (Don't forget to include the effects of any clamps or mass hangers you use.)

Weigh your mass with the triple-beam balance and compare the result to your calculation.

B. Meter stick supported away from center of gravity:

With only one weight suspended from one end of the meter stick, move the center support away from the center of the meter stick until you find a rotational equilibrium.

Calculate the torque about the pivot due to the single mass hanging from the stick.

The single weight is being balanced by the weight of the meter stick, which acts as if it were all concentrated at the center. Pretend that the meter stick is in fact massless. Where would you need to hang a second mass, equal to that of the meter stick, so that its lever arm would be sufficient to balance the single weight? Calculate this lever arm (from the new position of the pivot). Compare its position to the position of the center of mass.

Questions

When two unequal masses are balanced on either side of the meter stick, what is the relationship between the ratios of the masses and the ratios of their lever arms?

Is it possible for there to be no net (total) force on an object (such as the meter stick) but still have a non-zero torque? Explain and give and example.

How does the triple-beam balance work?

A. Meter stick supported at center of gravity:

Mass of stick:		
Mass of left clamp: Weight=	Mass of center clamp Weight=	Mass of right clamp Weight=:
Position of Center of mass of stick:		

Two Known Weights Data Table (first equilibrium arrangement of weights):

Sketch (labeled)	Weights, Position		Lever Arms:	Results:
	W_1= W_2=	X_1 = X_2 =	r_1= r_2=	τ_{CCW}= τ_{CW}= % Diff:

Discussion of how the balance of the meter stick varies as the weights are moved:

Two Known Weights Data Table (alternative equilibrium arrangement of weights):

Sketch (labeled)	Weights, Position		Lever Arms:	Results:
	W_1= W_2=	X_1 = X_2 =	r_1= r_2=	τ_{CCW}= τ_{CW}= % Diff:

Additional Discussion:

Three Known Weights Data Table (first equilibrium arrangement of weights):

<table>
<tr><td rowspan="3">Sketch (labeled)</td><td colspan="2">Weights, Position</td><td>Lever Arms:</td><td>Results (include direction)</td></tr>
<tr><td colspan="2">W_1= X_1 =
W_2= X_2 =
W_3= X_3 =</td><td>r_1=
r_2=
r_3=</td><td>τ=
τ=
τ=</td></tr>
<tr><td>τ_{CCW}=</td><td colspan="2">τ_{CW}=</td><td>% Diff:</td></tr>
</table>

Three Known Weights Data Table (second equilibrium arrangement of weights):

<table>
<tr><td rowspan="4">Sketch (labeled)</td><td colspan="2">Weights, Position</td><td>Lever Arms:</td><td>Results:</td></tr>
<tr><td colspan="2">W_1= X_1 =
W_2= X_2 =</td><td>r_1=
r_2=</td><td>τ=
τ=</td></tr>
<tr><td>W_3=</td><td colspan="3">Predicted position:</td></tr>
<tr><td colspan="2">Measured position:</td><td colspan="2">% Diff:</td></tr>
</table>

Discussion and Observations:

Unknown Mass:

Sketch (labeled)	Weights, Position	Results:
	W_1= ??? X_1 = W_2= X_2 =	W_1 (calculated) W_1 (measured)

Describe Calculations needed to determine unknown mass (here or on additional page)

B. Meter stick supported away from center of gravity:

Mass of stick (previously measured)
Position of Center of mass of stick (previously measured)
Position of pivot:

Sketch (labeled)	Weight, Position	Lever Arm: (from pivot)	Torque:
	W_1= X_1 =	r_1=	τ=
	Lever arm required for weight equal to stick to balance single weight:	Position required for weight equal to stick:	
Compare predicted position to center of mass			

Calculations and Discussion:

11. Rotational Inertia

Introduction

In the case of linear motion, a net non-zero force will result in linear acceleration, given by Newton's Law **F**=m**a** . The moving object will posses kinetic energy, K= ½ mv^2. For rotational motion, an unbalanced, non-zero torque, τ, will result in angular acceleration, α, according to the equation $t = I a$, where I is the rotational inertia of the object, and the rotating object will have rotational kinetic energy, K= ½ I ω^2 The rotational inertia (or moment of inertia) depends not only on the mass but also on how the mass is distributed about the axis of rotation.

Equipment

Rotational Platform	Masses, 50 gm mass hanger	Rods, clamps, 2 pulleys
Solid Disc, hollow disc	String	2 meter stick, stopwatch

For the class as a whole: scissors, level, balance, scale for heavy weights, verniers

Theory

Variously shaped objects are placed on the rotational apparatus. A system of pulleys are used so that a mass, m, can be attached to a string which is wound around the small radius of the rotational stand. A hanging mass produced tension in the string which ultimately applies a torque to the rotational apparatus and whatever is placed on top of it. We can measure rotational inertia by looking at either the angular acceleration of objects placed on the rotating platform or by looking at the increase in rotational kinetic energy. For the hanging mass, Newton's Law predicts:

$$F = mg - T = ma$$

while for the rotating platform the torque is given by:

$$t = r_\perp F = r_\perp T = Ia$$

where $r_\perp$ is the radius of the part of the platform which the string is wrapped around. If the string unwinds without slipping, then we also have the relationships:

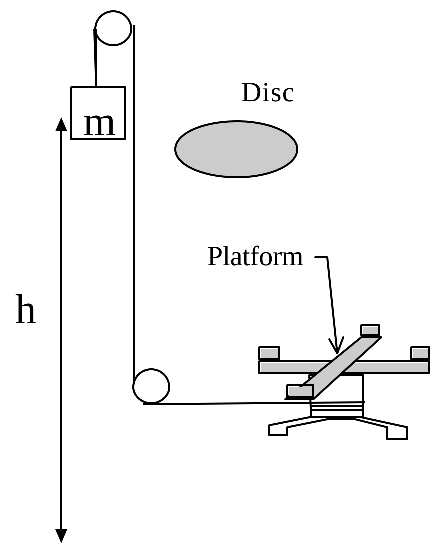


Figure 9: Rotational Apparatus

$$a = a / r_\perp \qquad w = v / r_\perp \;\; \omega = v / \; r_- \qquad q = s / r_\perp$$

where a and v are the acceleration and velocity of the string as it unwinds, $a, w,$ and q are the angular acceleration, angular velocity and angle in radians of the platform as it turns, s is the arclength of the circle (which is equal to the length of string unwound) and $r_\perp$ is the radius about which the string is wound.

In this lab we will relate the linear motion of the falling mass to the angular motion of the platform. This requires the equations for constant acceleration:

$$\Delta x = v_0 t + \tfrac{1}{2} a t^2 \qquad v = v_0 + at \qquad v^2 = v_0{}^2 + 2a\Delta x \quad \text{and} \quad g = 9.8 \frac{m}{\sec^2}$$

Alternatively if we apply Conservation of Energy then the gravitational potential energy of the mass (which it possesses due to its initial height h) is converted to kinetic energy of both the falling mass and the rotating platform. Before the mass falls (if the system starts from rest) K=0 and all the energy is potential E= mg h. After the mass falls from a height h to height of zero, all of the energy is kinetic and $E= ½ mv^2 + ½ I \omega^2$. If there is no friction then E is a constant and $mgh = ½ mv^2 +½I\omega^2$.

Procedure

Note: guide the string as you wind it around the pulley to avoid tangling. Make sure it winds around the pulley and does not become caught inside around the small central post. If the string becomes caught in the central post of the rotation platform then turn the platform upside down to take its weight off the string --- the string can then be more easily unwound.

Level the rotational platform and adjust the pulleys so that the string pulls horizontally on the platform and the mass can fall vertically.

Measure the radius of the part of the apparatus which the string is wrapped around and the height which the mass will fall to the floor. Record as $r_{-}=R_{string}$ and h.

When the string and pulleys are all in place, suspend mass, M_0, from the string until it is just enough to produce slow uniform motion (constant speed). This will be just enough to counterbalance any frictional forces. In later parts of the experiment this weight will remain on the hanger but not be counted as part of the hanging mass. Record this data in the table as the “Zero Acceleration” weight $W_0 =M_0g$.

Now add about 50 grams additional mass (or enough to produce significant acceleration) and measure the time that it takes the mass to fall from rest until it hits the floor. *For good results the time to fall should be at least 5 seconds*. Make three trials making sure to use the same initial height each time and average your results.

Assume constant acceleration and calculate the acceleration, a, of the mass from the known initial height, initial velocity and time for fall.

Calculate the angular acceleration, a , of the rotating platform assuming the string did not slip.

Once you know the acceleration of the mass you know the net force on the mass. Draw a force diagram of the falling mass and calculate the Tension, T, in the string (do not include the “Zero Acceleration” weight W_0 in this calculation, do include the added weight, W).

Draw a force diagram for the rotating platform (using an overhead view) and calculate the torque applied to the platform by the tension, T, in the string.

Find the rotational inertia of the platform, I_0.

<u>Rotational Inertia of objects added to the platform</u>

Now you will add objects on top of the platform and determine their rotational inertia. If the new object has rotational inertia I_2 then the total rotational inertia of both the platform and the object on top of it is $I_{total} = I_0 + I_2$. You will measure I_{total} then subtract I_0 (as determined above) to find the rotational inertia of the second object I_2.

Perform the following measurements for the solid disc, the hollow ring and the bar or small cylinder. (The latter can be placed on top of the disc if necessary, in which case include the rotational inertia of the disc as part of I_0).

Make a sketch of your object and record all of its dimensions and mass. Place the new object on the platform, making sure to align its center with the axis of rotation.

Repeat the measurement of the base weight W_0 since the forces of friction will change.

Use the procedure outlined above to determine I_{total}. Subtract your previous value for I_0 to find the rotational inertia of the object you added.

Calculate the percent difference between the rotational inertia you measure and the theoretical value (See Table).

Rotational Inertia of various objects about an axis through the center of mass

Solid Disc or cylinder, radius R	Hollow disc, inner radius R_2, outer radius R_1	Bar, Length A and Width B
$I = ½ MR^2$	$I = ½ M (R_1^2 + R_2^2)$	$I = 1/12 M (A^2 + B^2)$

Additional Exercises

Rotational Inertia about axis parallel to center of mass

The parallel axis theorem states that the rotational inertia of an object about any axis is equal to the rotational inertia about a parallel axis through the center of mass plus the mass times distance between the axis squared. $I = I_{CM} + Mh_2$. (Note this h is not height!) Place an object off-center on the platform and determine the rotational inertia about this new axis. Make a sketch of the arrangement, label all dimensions and confirm the parallel axis theorem.

Conservation of energy

For one set of data show that energy is conserved as the mass falls (decreasing its potential energy) and both the mass and rotating platform accelerate (increasing their kinetic energy).

Questions:

Calculate the ratio of the mass of the disc to the mass of the ring. Calculate the ration of the rotational inertia of the disc to the rotational inertia of the ring. Compare these two ratios. If they are not equal then rotational inertia must not depend on mass only. What other factors effect rotational inertia? Discuss.

Compare the moments of inertia of the different objects. <u>Discuss how they differ in their physical dimensions, masses, and how the mass is distributed</u>. Do the measured values for rotational inertia vary as you would predict?

How do your moments of inertia compare to the theoretical predictions? Discuss possible sources of error or uncertainty. (Do your objects match their theoretical descriptions? How big of an error is any difference?)

Why doesn't rotational inertia depend on the height (thickness) of the objects?

Compare the rotational inertia for rotating on-center and off-center. Does it vary as you would expect?

Rotational Inertia of Platform (Cross)

Sketch:	Initial Height: Radius $r_\perp = R_{string}$: Mass needed for "Zero Acceleration": M_0= Weight W_0 =
Mass added to M_0: Weight added, W:	Times measured: 1) 2) 3) Average:

Force Diagrams and Calculations. Work out on scratch paper, then write clearly, show all steps and label and discuss each.

Results of Calculations:

Accel of mass, a	Angular accel. α	Tension in string, T	Lever-arm for torque	Torque, τ	Rotational Inertia, I_0

Rotational Inertia of Other object:

Description of this Object and procedure used:

<table>
<tr><td>Sketch:</td><td rowspan="2">Initial Height:

Radius $r_{\perp}$ =R_{string}:

Mass needed for “Zero Acceleration”:
M_0=
Weight W_0 =</td></tr>
<tr><td>Mass: Dimensions:</td></tr>
<tr><td>Mass added to M_0:

Weight added, W:</td><td>Times measured:
1)
2)
3)
Average:</td></tr>
</table>

Results of Calculations:

Accel of mass, a	Angular accel. α	Tension in string, T	Lever-arm for torque	Torque, τ	Rotational Inertia, I_{total}	I_0	I_2 for this object
Experimental Value for I_2 (from above)			Theoretical Value for I_2			% Diff:	

Discussion and Calculations. Work out on scratch paper then write neatly and label or discuss all parts. Include extra paper or write on back if needed.

Rotational Inertia of Other object:

Description of this Object and procedure used:

<table>
<tr><td>Sketch:</td><td rowspan="2">Initial Height:

Radius $r_{\perp}$ =R_{string}:

Mass needed for “Zero Acceleration”:
M_0=
Weight W_0 =</td></tr>
<tr><td>Mass: Dimensions:</td></tr>
<tr><td>Mass added to M_0:

Weight added, W:</td><td>Times measured:
1)
2)
3)
Average:</td></tr>
</table>

Results of Calculations:

Accel of mass, a	Angular accel. α	Tension in string, T	Lever-arm for torque	Torque, τ	Rotational Inertia, I_{total}	I_0	I_2 for this object
Experimental Value for I_2 (from above)			Theoretical Value for I_2			% Diff:	

Discussion and Calculations. Work out on scratch paper then write neatly and label or discuss all parts. Include extra paper or write on back if needed.

Rotational Inertia of Other object:

Description of this Object and procedure used:

<table>
<tr><td>Sketch:</td><td rowspan="2">Initial Height:

Radius $r_{\perp}$ =R_{string}:

Mass needed for “Zero Acceleration”:
M_0=
Weight W_0 =</td></tr>
<tr><td>Mass: Dimensions:</td></tr>
<tr><td>Mass added to M_0:

Weight added, W:</td><td>Times measured:
1)
2)
3)
Average:</td></tr>
</table>

Results of Calculations:

<table>
<tr><td>Accel of mass, a</td><td>Angular accel. α</td><td>Tension in string, T</td><td>Lever-arm for torque</td><td>Torque, τ</td><td>Rotational Inertia, I_{total}</td><td>I_0</td><td>I_2 for this object</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="3">Experimental
Value for I_2 (from above)</td><td colspan="3">Theoretical
Value for I_2</td><td colspan="2">%
Diff:</td></tr>
</table>

Discussion and Calculations. Work out on scratch paper then <u>write neatly and label or discuss all parts</u>. Include extra paper or write on back if needed.

Rotational Inertia of Other object:

Description of this Object and procedure used:

<table>
<tr><td>Sketch:</td><td rowspan="2">Initial Height:

Radius $r_{\perp}$ =R_{string}:

Mass needed for “Zero Acceleration”:
M_0=
Weight W_0 =</td></tr>
<tr><td>Mass: Dimensions:</td></tr>
<tr><td>Mass added to M_0:

Weight added, W:</td><td>Times measured:
1)
2)
3)
Average:</td></tr>
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Results of Calculations:

<table>
<tr><td>Accel of mass, a</td><td>Angular accel. α</td><td>Tension in string, T</td><td>Lever-arm for torque</td><td>Torque, τ</td><td>Rotational Inertia, I_{total}</td><td>I_0</td><td>I_2 for this object</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="3">Experimental
Value for I_2 (from above)</td><td colspan="3">Theoretical
Value for I_2</td><td colspan="2">%
Diff:</td></tr>
</table>

Discussion and Calculations. Work out on scratch paper then <u>write neatly and label or discuss all parts</u>. Include extra paper or write on back if needed.

Chapter 3: Thermodynamics and Fluids

12. Specific Heats and Latent Heats

Introduction

Heat is the transfer of energy due to differences in temperature. Exactly how much the temperature changes due to transfer of heat depends on the material in question. For example, it takes about three and a half times as much energy to change the temperature of iron as it does to change the temperature of the same mass of lead by the same amount. The relationship between heat added (or removed), Q, and change in temperature, ΔT, is given by:

$$Q = m\, c\, \Delta T$$

where Q is the amount of heat, m is the mass, ΔT is the change in temperature and c is the *specific heat* of the material. In general the specific heat may change with temperature and will change as the phase (solid, liquid or gas) of the material changes. In addition to the heat required to change temperature, additional energy is required when a material changes *phase*. The energy per unit mass required to change the nature of the bonding in the substance as it changes from solid to liquid or from liquid to gas is called the *Latent Heat,* and is given by

$$Q = L\, m,$$

where, again, Q is heat added or removed, m is mass and L is the latent heat. Latent heat for freezing/melting is called the Latent heat of fusion, L_F and latent for boiling/condensing is called the Latent heat of vaporization, L_V.

Equipment

Balance	3 styrofoam cups (8oz.)	metal specimens (5ea.)	specimen tongs
metal beaker (for boiling)	beaker tongs	water	beaker (250ml)
hot plate	2 thermometers	liquid nitrogen*	stop watch

*(*Note for teacher: Liquid Nitrogen in Bosshart 318, key 110*)

A. Mixing Warm and Cool Water (a "warm-up" exercise)

Determine the mass of a "triple" styrofoam cup (i.e. stacked one inside the other). (Record your data in Table A.) Put about 100 grams of unheated water (i.e. 100ml since by definition 1 cubic centimeter (1/1000 liter) of water has a mass of 1g!) and measure the mass of the cups plus the water. Measure the water temperature. Be sure to stir the water with the thermometer until you are sure the thermometer is in thermal equilibrium with the water. Heat about 50ml of water to about 45°C. Remove the beaker of warm water from the hot plate and measure the water temperature, then immediately pour the warm water into the cup with the unheated water. Stir thoroughly and measure the final temperature. Record the total mass of the cup with the mixed water.

Write up a description of your procedure and record the masses and temperatures of the unheated, heated, and mixed water samples in a data table.

Does the final temperature you measured seem reasonable? Explain, using the word “heat” in your explanation.

Would you necessarily expect the same result if you mixed water with alcohol (for example?) Explain why or why not.

Set up an equation using $Q = m\ c\ \Delta T$ to express the hypothesis that the heat gained by the cool water during mixing is equal to the heat lost by the warm water. Solve this equation for the final temperature of the mixture and substitute the measured values for the initial temperatures of the warm and cool water, and calculate the expected final temperature of the mixture. How does your calculated result compare to your measurement of the final temperature?

B. Hot Metal in Cool Water: Specific Heats

Fill a 250 ml beaker about half full of water, pour the water into the metal beaker, place it on a hot plate and begin heating it. Turn the hot plate knob to the highest setting. CAUTION: Be careful handling hot beakers, water, metals or hot plate surfaces. Gloves and safety goggles are available for your protection.

While waiting for the water to boil, measure and record the total mass of 5 specimens of the metal you are using (in Table B), then carefully place them in the water being heated using specimen tongs

Empty out the triple-cup "calorimeter" and fill it half full with room temperature water. Measure and record the total mass of the cups plus the water and the temperature of the room temperature water in Table B.

Leave the metal specimens in the hot water for at least 5 minutes after boiling has begun so you can assume that the temperature of the metal specimens is the same as the temperature of the boiling water.

Using the specimen tongs, carefully remove each metal specimen from the boiling water and quickly lower it into the room temperature water in the styrofoam cup. (All specimens should be covered with water.)

Gently stir the water in the styrofoam cups for several minutes, frequently checking your thermometer for its highest reading. When the thermometer reading starts to fall, record the highest temperature reached in Table B as the final temperature of the system.

Write a paragraph explaining your procedure and discussing your data.

Repeat steps 1 through 7 for a set of specimens of another metal, recording your information in the data table.

Explain qualitatively what is happening to account for the final temperature.

Assume the heat gained by the water equals the heat lost by the specimen. The specific heat of water is 4186 J/(kg$\cdot$ ^{0}C). Explaining your work in full, calculate the specific heats of Aluminum and Copper. Compare to known values by finding a percent difference. Record the results in a table and discuss reasons for discrepancies.

C. Latent Heat of Liquid Nitrogen

The rate of evaporation of liquid nitrogen

Pour about 180-200 grams of liquid Nitrogen in two Styrofoam cups, one inside the other. The liquid nitrogen will be boiling at room temperature so the temperature inside the liquid will be the boiling point.

Measure the precise mass using a balance scale. Immediately start a stopwatch and time how long it takes for 10 grams to boil away. A convenient way to do this is to set your balance for 10 grams lower and wait until the balance arm swings back up. Record this data as Time = _________ for 10 grams to boil away.

Calculate the rate of evaporation of the liquid nitrogen: that is the grams per second boiled away. Record this as Rate of evaporation = _________ grams/sec.

The Latent heat of vaporization of liquid nitrogen

Make two double-cup containers and label them "A" and "B". Carefully measure the mass of each container. Record Mass of A and Mass of B.

Pour 100 grams of warm (~50 °C) water in cup "A". Measure the mass and the temperature of the water. **REMOVE THE THERMOMETER.** Record Mass of Water and Initial Temp. of Water.

Pour about 40 grams of liquid nitrogen in cup "B". Quickly (to minimize loss due to evaporation) measure the mass of cup B (containing liquid nitrogen) and then pour the liquid nitrogen into cup A with the warm water. A white cloud of nitrogen vapor will show that the liquid nitrogen is boiling away. *Measure the time to evaporate all the nitrogen*. Record Mass of Nitrogen (not including cup) and Time to Evaporate.

When all the nitrogen is gone, gently stir the water until all the ice has melted. Measure the temperature of the water. Record as Final Water Temperature.

Determine how much nitrogen would have evaporated in the same time even without the water. (Use your rate from the previous part.) Subtract that from the initial mass of nitrogen to find out how much evaporated due to the water only.

Assume the heat released as the water cooled went into boiling the nitrogen. Determine the Latent heat of vaporization for nitrogen.

Write out a description of your procedure and record your data in a data table.

Describe your calculations in full, including a verbal description of what is happening.

Compare to the accepted value for L_V of nitrogen. The results may not be precise, since this experiment is not sufficiently isolated from the room, but you should be able to determine the order of magnitude of the latent heat of vaporization.

Table 1

Material (Room Temp)	Specific Heat c, $Joules/kg\cdot{}^0C$
Aluminum	900
Brass	380
Copper	387
Iron (or steel)	448
Lead	128
Glass	837
Water (15 ^{0}C)	4186
Ice (-5 ^{0}C)	2090

Table 2

Material	**Melting Temp ^{0}C**	**L_F J/kg**	**Boiling Temp ^{0}C**	**L_V J/kg**
Nitrogen	-209.65	$2.55x10^4$	-195.81	$2.01x10^5$
Water	0.00	$3.33x10^5$	100.00	$2.26x10^6$
Ethyl Alcohol	-114	$1.04x10^5$	78	$8.54x10^5$

A. Mixing Warm and Cool Water:
Procedure:

Volume of Cool Water	Mass of Cool Water	Temp. Cool Water	Volume of Warm Water	Mass of Warm Water	Temp Warm Water	Final Temp. of Mixture

Discussion:

B. Hot Metal in Cool Water
Procedure:

Material	Mass of specimen	Temp. of specimen	Mass of Water	Initial Temp. of Water	Final Temp.	Specific Heat	% Diff.

<u>Discussion and Calculations. Add a separate page</u>

C. Liquid Nitrogen
Evaporation of liquid nitrogen
Procedure:

Time = ____________________ for 10 grams to boil away.

Rate of Evaporation = ________________ grams/sec

Latent heat of vaporization
Procedure:

Mass Water	Initial Temp Water	Final Temp Water	Heat Lost by Water	
Initial Mass Nitrogen	Time to Evaporate	Amount that would have Evaporated by itself	Mass Evaporated by Water	Heat added to Nitrogen by Water
Latent Heat of Nitrogen		Accepted Value		% Difference

<u>Discussion and Calculations. (Add a separate page)</u>

13. Coefficient of Linear Expansion

Introduction

In this experiment, you will study the thermal expansion properties of various materials. With few exceptions materials expand somewhat when heated through a temperature range that does not produce a change in phase (melting, freezing, boiling etc.). The added heat increases the average amplitude of vibration of the atoms in the material, which increases the average separation between the atoms. Although this effect is small, it is very important in any application that involves using different materials in an environment where they are heated and cooled. For example, if a rivet of one metal is used inside a hole in a different material, it can become too tight or too loose if the thermal expansion of the materials is very different.

Equipment

linear expansion apparatus; rods of different materials; boiler, hot plate, tubing
tongs, gloves, funnel, eye protection, paper towels; thermometer, 400 ml beaker

Note: heating is time consuming -- set hot plate on high and heat water right away!

Theory

For solids, the material undergoes thermal expansion as a whole: that is its volume expands. For materials that are not isotropic (that is uniform in all directions),such as an asymmetric crystal for example, the thermal expansion can have a different value depending on the axis along which the expansion is measured. Thermal expansion can also vary somewhat with temperature so that the degree of expansion depends not only on the magnitude of the temperature change, but on the absolute temperature as well.

Suppose an object of length L undergoes a temperature change of magnitude ΔT. If ΔT is reasonably small the change in length, ΔL, is generally proportional to L and to ΔT. Stated mathematically, ``proportionally" means that:

$$\Delta L = a \cdot \mathrm{L} \cdot \Delta \mathrm{T}$$

where a (Greek letter “alpha”) is called the *coefficient of linear expansion* for the material.

For an isotropic material, a will be the same in all directions, so we can measure a simply by measuring the change in length of the material. The values obtained for the coefficient of linear expansion will be compared with accepted values to determine the composition of each rod.

Substance	**Coefficient of linear expansion,** a $(\times 10^{-6}\ /^{0}C)$	**Substance**	**Coefficient of linear expansion,** a $(\times 10^{-6}\ /^{0}C)$
Aluminum	24.0	Nickel	12.8
Brass	18.9	Silver	18.8
Copper	16.8	Steel	13.2
Glass (common)	8.5	Tin	26.9
Iron	11.4	Zinc	26.3
Lead	29.4		

Procedure

.In this lab you will measure a for rods made of different materials. These metals are isotropic so that a need only be measured along one dimension. Within the limits of this experiment, a does not vary with temperature.

To make this measurement the hollow rod is placed in the apparatus. The length of the tube is measured at room temperature, then steam is passed through it. The expansion of the metal is measured using the built in dial-gauge.

BEWARE: THE ROD AND JACKET WILL BECOME VERY HOT. BE EXTREMELY CAREFUL!

Fill the boiler about one-half full of water, cap it, and connect the rubber hose to its spout. Heat it on the hot plate. While waiting for the water to boil, the rest of the apparatus can be assembled

First, the length of the rod L must be measured using a ruler.

Make an estimate of the *uncertainty* in the length and call this number s_L.

Next, insert the rod into the aluminum jacket and put it in place on the supporting apparatus. It is very important that the rod is properly placed. One end must be firmly pressed against the screw protruding from the supporting apparatus, while the other must be in contact with the micrometer probe. The rod must not be bowed however. The micrometer will be used to measure ΔL .To zero its reading initially, simply turn the face of the meter until the needle points to zero.

Insert the thermometer in the central opening in the aluminum jacket until it is barely touching the rod. Record the initial temperature T_{initial} . Make note of the *uncertainty* in your temperature measurements.

Now, it is time to heat the rod. Connect the free end of the rubber hose to the opening in the jacket that is sticking upwards. Align the whole apparatus so the the remaining opening (directed downwards) is situated over the sink. This opening allows steam and water to escape the jacket. The temperature of the rod should remain essentially unchanged until the water begins to boil. At this point, steam will enter the jacket through the rubber tube and heat the rod.

While the rod is heating, observe the micrometer and make a note of how many full revolutions it makes.

Wait until the rod has reached thermal equilibrium with the steam. In other words, wait until the temperature of the rod is no longer increasing and has remained constant for a couple of minutes. At this point, record the final temperature T_{final}.

Read the micrometer to obtain ΔL . Note that each minor tick on the micrometer scale represents 0.01 mm and a full revolution by the needle represents a length change of 1.00 mm. Be sure you observe each time that the micrometer completes a full revolution, since you must then add 1.00 mm to the final reading for ΔL .

Make an estimate of the *uncertainty* in the change of length and call this number $s_{\Delta L}$

Calculate the change in temperature, Estimate the uncertainty in change of temperature by adding the uncertainty in the initial and final temperatures. Call the total uncertainty in temperature $s_{\Delta T}$

After obtaining the necessary measurements, unplug the hot plate.

Next, the rod just measured must be cooled. This is done by removing the thermometer or the tube from the opening. Then, a funnel is inserted into the opening and cool water is poured into it. This water will exit through the hole situated over the sink. Tilting this end upwards temporarily will allow the water to flow throughout the length of the jacket and cool all portions of the rod.

Once the rod is cool, remove it and insert another rod.

Repeat the experiment for the three remaining rods.

CALCULATIONS

Use the measured values of L, ΔT, and ΔL to calculate $a_{\exp erimental}$.

Calculate the uncertainty in your determination of a. The equation for a is given by $\Delta L = a \cdot \mathrm{L} \cdot \Delta \mathrm{T}$. When multiplying or dividing uncertain numbers, the *fractional* uncertainties (s_L/L for example) are squared and summed that give square of the net uncertainty, thus., $s_a = a\sqrt{\left(\frac{s_L}{L}\right)^2 + \left(\frac{s_{\Delta L}}{\Delta L}\right)^2 + \left(\frac{s_{\Delta T}}{\Delta T}\right)^2}$. Which measurement has the greatest effect on the final uncertainty?

Compare the experimental values for the coefficient of linear expansion with those found in the table. Use these numbers (and any other observations that you can make) to determine the composition of each rod. Take into account the uncertainty in your measurements to see if your result is conclusive.

Calculate the quantity $\left|\frac{a_{\exp eriemental} - a}{s_a}\right|$. This is a measure of how accurate your experiment is. If this is less that one you are doing pretty well, if less than 2 you are doing okay, if greater , then hmmmmm.......

If the final results seem outside the experimental uncertainties, discuss any random or systematic errors.

See Appendix for further discussion of uncertainty.

Description of rod:

Length, L		T_{initial}		ΔT	
s_L		T_{final}		$s_{\Delta T}$	
Full turns		Additional			
ΔL		$s_{\Delta L}$			

Calculations:

Coefficient, a ________________ Uncertainty, s_a ________________

Probable type of rod: ________________________________

Description of rod:

Length, L		T_{initial}		ΔT	
s_L		T_{final}		$s_{\Delta T}$	
Full turns		Additional			
ΔL		$s_{\Delta L}$			

Calculations:

Coefficient, a ________________ Uncertainty, s_a ________________

Probable type of rod: ________________________________

Description of rod:

Length, L		T_{initial}		ΔT	
s_L		T_{final}		$s_{\Delta T}$	
Full turns		Additional			
ΔL		$s_{\Delta L}$			

Calculations:

Coefficient, a ________________ Uncertainty, s_a ________________

Probable type of rod: ________________________________

14. The Ideal Gas Law

Introduction

The ideal gas law,

$$PV = nRT,$$

is a useful formula that comes from experimental observation. In this formula P stands for pressure in N/m^2 (i.e. Pascals); V is the volume in cubic meters; n is the number of moles; R = 8.31 J/mol K is the universal gas constant; and T is the absolute temperature in Kelvin. This law has been tested on numerous gases and shows a remarkable agreement with experiment for a large range of pressures, volumes and temperatures when the number of moles per unit volume is not too high.

Equipment

- Boyle's Law unit
- metal sphere
- 1500-ml beaker
- hot plate
- thermometer
- graph paper
- stopcock lubricant
- ice

Theory: Boyle's Law

When T and n in the ideal gas law are kept constant the variation of pressure with volume is called Boyle's law.

<u>Notes on the operation of the Boyle's Law unit:</u>

The pressure gage employs a Bourdon tube, a circular, sealed-end tube having an oval cross section that tends to "uncurl" when the internal pressure exceeds the external pressure. Observe the movement of the tube, linkage and pointer when you vary the pressure.

To operate the "quick disconnect" fitting, slide the brass ring at the base of the pressure gage about 5 mm toward the body of the gage and pull the male fitting.

Wait about 5 seconds after setting the desired volume with the plunger before reading the pressure. Usually a slight initial drop in pressure will be noted. This results from the gas returning to room temperature after a slight temperature increase due to compression.

Before making a reading, lightly tap at the base of the gage with your finger to relieve friction effects in the gage gear and linkage system.

The gage readings are in pounds per square inch (psi); 1 psi = 6895 Pa. Normal atmospheric pressure is 14.7 lb./ in^2. If your gage does not read 14.7 psi when open to atmospheric pressure, you must correct all of your readings by either adding or subtracting the difference between your reading of atmospheric pressure and 14.7 psi. (If your gage is grossly in error, bring it to the attention of your instructor. It may be necessary to remove the plug directly behind the gage and adjust the pointer using a screwdriver.)

If the pointer in the gage starts to drift when the volume is held constant, the gas may be leaking around the syringe plunger. You may coat it lightly with stopcock lubricant to prevent leakage

Procedure: Boyle's Law

In Boyle's law the volume V is the total volume occupied by the gas, thus $V = V_s + V_o$ where V_s is the volume of the syringe and V_o is the volume of the gage, tubing and fittings. To obtain V_o follow this procedure:

Disconnect the tubing at the gage and set the plunger at the 0-cc mark on the syringe. Reconnect the tubing and record the pressure reading.

Pull the plunger in the syringe until the pressure reading is 1/2 of the original reading. According to the ideal gas law, the total volume at this pressure is must be twice V_o. Therefore, V_s=V_o=________cc. Note that this value for V_o must be added to the volume of the syringe at all pressure settings.

Disconnect the syringe from the gage; set the plunger at 15 cc and reconnect. Move the plunger in increments of 5 cc to all values above and below 15 cc. At each value record pressure and volume. Repeat this three times and take averages.

Plot P vs. 1/V and analyze the graph.

To what extent is it consistent with Boyle's law? (i.e. is the graph a straight line through the origin?)

Finding the Absolute Zero of Temperature

(note: This will not require a trip to Antarctica.)

The third law of thermodynamics states that absolute zero is an unattainable limit of low temperature. At this temperature all motion should cease, and the pressure of the ideal gas should be zero. We can neither get to zero pressure nor to very low temperatures in this lab, but we can extrapolate our results to those values and get a decent estimate of the absolute zero of temperature.

The ideal gas law, with temperature, T_C, in degrees Celsius, is

$$PV = nR(T_C + T_0),$$

where T_0 is the absolute temperature, in K, at 0 °C. Thus, for a constant number of moles, n, and a constant volume, V, of gas, the temperature is linearly related to the pressure:

$$T_C = m\,P - T_0,$$

where the slope of the graph is

$$\text{slope} = \text{"m"} = V / nR\ .$$

Procedure: Absolute Temperature

Connect the tubing from the metal sphere to the pressure gage.

The six 1500-ml beakers should be 3/4 full with water at different temperatures. One should contain a mixture of ice and water, in equilibrium at 0 °C. A second beaker should have water at room temperature. The remaining 4 beakers should be maintained at nominal temperatures of 40, 58, 77 and 95 °C. Each lab group should maintain a beaker at a different temperature. Consult your instructor or the other lab groups before selecting your temperature. IMPORTANT: Do not allow the plastic tubing to come in contact with the heat source.

Starting from the lowest temperature beaker, immerse the sphere in the water and record the pressure and temperature (below) once the sphere is in equilibrium with the water (it may take a couple of minutes). Proceed to the hotter temperature beakers. USE CAUTION WHEN WORKING WITH HOT WATER. EYE PROTECTION AND GLOVES ARE AVAILABLE FOR YOUR PROTECTION.

Using Kaleidagraph or Excel, plot temperature vs. pressure and find the equation of the line for your data. From the equation of the line determine the absolute zero of temperature in °C (i.e. the temperature, in Celsius, at which P = 0). A good result would be within 5% of the accepted value of -273 °C.

Plot T_C vs. P and analyze the graph.

From your analysis, determine the absolute temperature at O^0C.

Boyle's Law

$V = V_S + V_O$	$1/V$	P_1	P_2	P_3		P_{AVE}

<u>Plot P vs. 1/V and analyze the graph.</u>
To what extent is it consistent with Boyle's law? (i.e. is the graph a straight line through the origin?)

Finding the Absolute Zero of Temperature

T_0						
P						

<u>Plot T_C vs. P and analyze the graph.</u>
From your analysis, determine the absolute temperature at O^0C.

Linear equation of graph: ____________________________

Absolute zero temperature: T_0 = _________°C

15. Buoyancy and the Density of Liquids

Introduction

Which weighs more, a pound of lead or a pound of feathers? If your answer to this question is " a pound of lead", then you are confusing "weight" with "density." If you have a pound of anything, it weighs a pound (i.e. it is attracted by Earth's gravity with a force of one pound). The density of a substance is its mass divided by its volume. Since a pound of lead will not take up as much space (i.e. volume) as a pound of feathers, it will have a greater density. Density is one of the most important properties by which we characterize a substance. In this lab you will measure the density of various objects and of a liquid by measuring buoyant force, displaced volume, and mass and volume directly.

Equipment

- assorted metal cylinders
- irregular solid
- "unknown" liquid
- displacement can
- overflow can
- specific gravity bottle funnel
- vernier caliper
- ruler
- string & scissors
- triple beam balance
- small beaker (50 ml)
- beaker (200 ml)
- graduated cylinder (10 ml)

Theory

Density depends on how closely spaced the atoms of a substance are as well as on the mass of each atom, and therefore is a physical property of the substance. Its numerical value depends on the units used for mass and volume. For example, the proper SI unit for density is kilograms per cubic meter (kg/m^3). A unit that is often more convenient for use in the laboratory is grams per cubic centimeter (g/cm^3 or g/cc), or equivalently, grams per milliliter (g/ml), since 1 milliliter = 1 cubic centimeter of volume. This unit is particularly handy, since the density of water, the must commonly encountered liquid, is 1.0 g/cm^3 (not just by coincidence). If we measure density in one of these units we immediately know if a substance is more dense or less dense than water, and hence whether a solid "chunk" of it will float or sink when placed in water. In fact, these units are so commonly used that they are often omitted, in which case we speak of the "specific gravity" of a substance, which is numerically equal to the ratio of the density of the substance to the density of water. In other words, the specific gravity of a substance is numerically equal to its density expressed in g/cm^3, but without the units.

Archimedes' principle states that the buoyant (i.e. upward) force on an object due to the fluid in which it is immersed is equal to the weight of the fluid displaced by the object. This is why a solid immersed in a liquid, for example, appears to weigh less, and why some things float. This principle will be used to find the specific gravity of an object immersed in water.

Procedure

1. Hold a cylinder of a relatively high density metal, such as steel, brass, or lead in one and a cylinder of a lower density metal, such as aluminum in the other hand, and get a qualitative feel for their relative weights. Using a common ruler, make approximate measurements of the lengths and diameters of the cylinders. Using a vernier caliper, make more precise

measurements of these dimensions (see Appendix: Reading a Vernier). Compute the volumes of the cylinders using these measurements.

2. Use a laboratory balance to determine the masses of the two cylinders, then compute the density of each by dividing its mass, in grams, by its volume, in cm^3. Record your measured and calculated values. (How do they compare to your qualitative feel for the masses and densities?)

3. Tie a string around one of the cylinders and lower it into a can of water. As the cylinder becomes immersed in the water, notice the change in its apparent weight (i.e. the pull of the string) and the change in the water level in the can.

4. Pour water into an overflow can (Figure A) until it slowly flows out of the spout. Stop pouring and wait until the spout stops dripping. Measure the mass of a graduated cylinder then place it under the spout and slowly lower the cylinder into the water, catching the overflow in the graduated cylinder. Determine the mass of the overflow water and thereby determine its volume, based on the known density of water.

5. Determine the density of the cylinder using the volume of the displaced water and its previously measure mass. Compare this value with that determined in step 2. What is the specific gravity of the cylinder?

6. Arrange a laboratory balance and a vessel of water as shown in Figure B. Suspend the cylindrical mass from the underside of the balance by attaching the string to the balance arm and measure its mass. If the mass of the cylinder is not the same as was determined in step 2, use the zero adjustment on the balance to make it read the same. Now measure the mass of the suspended cylinder while it is totally submerged in water (but not touching the bottom of the can). The difference between this reading and the mass measured in air is due to the buoyant force of the water. Does this mass difference correspond closely to any of your previous measurements? If so, which one? Is this to be expected based on Archimedes' principle?

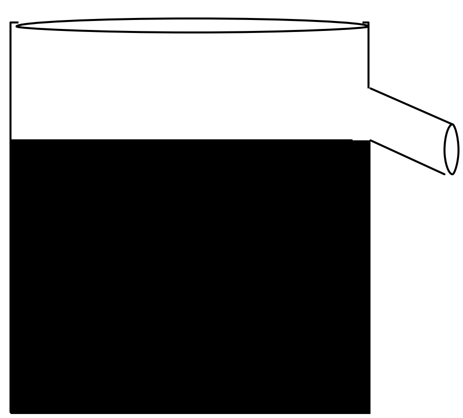


Figure A: Overflow Can

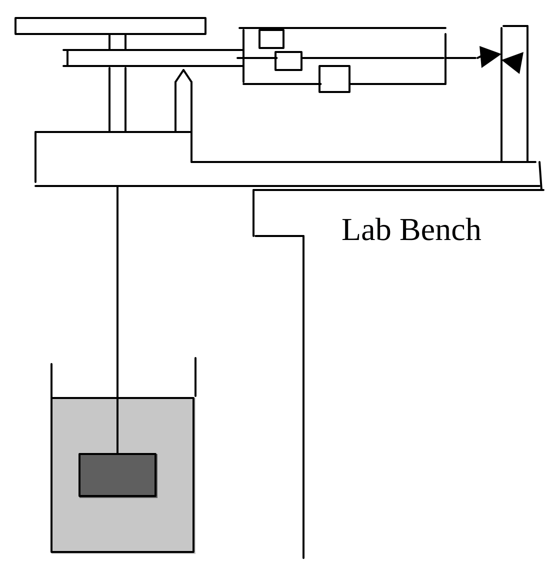


Figure B: Buoyant Force Arrangement

7. Compute the ratio of the mass reading of the cylinder (in air) to the difference between the mass readings obtained in air and submerged in water. To what does this correspond? Explain.

8. Do your observations in step 3 suggest a possible method for finding the volume of an irregularly shaped solid? Determine the volume of an irregularly shaped object and use the methods of steps 2 & 4 to determine its specific gravity. Next, use the method of step 7 to determine its specific gravity and compare results by calculating the per cent difference. Which method do you think gives the most accurate result? Why? Record your data and the kind of material of the irregular body.

9. **Specific Gravity of a Liquid by Specific Gravity Bottle (Pycnometer).** The specific gravity bottle is provided with a perforated glass stopper that permits the excess liquid to flow out of the bottle when the stopper is inserted. This results in a well defined volume of any liquid placed in the stoppered bottle. First, determine the mass of a clean, dry bottle and record. Then fill it with the "unknown" liquid by pouring very carefully from a small beaker. Make sure the outside of the bottle and the surface of the balance platform are dry and measure and record the mass of the bottle + liquid x. Pour liquid x back into the beaker and rinse the bottle with water. Then fill the bottle with water and measure and record the mass of the bottle + water as before. From the measured masses, calculate the mass of each equal volume of liquid and calculate the specific gravity of liquid x and record. (Note: The handbook value for the specific gravity of isopropyl alcohol is 0.78.)

10. Use the graduated cylinder and balance to determine the density of a liquid directly by mass and volume measurements. (The mass of the empty graduated cylinder, which must be subtracted from the mass of the graduated cylinder with the liquid in it, is commonly referred to as a "tare".) When reading the volume of the liquid in the cylinder, notice the "meniscus" shape of the liquid surface due to the attraction (or repulsion) between the liquid and the walls of the cylinder. Be sure to use the height of the center of the liquid column for your measurement, not the edge.

Questions

1. You determined the volume of a cylinder by two different methods. Which do you think is more accurate and why? Can you suggest yet another method for finding the volume?

2. Can you suggest a method of measuring the specific gravity of a solid that's simpler than using the displacement can? If so describe it.

3. From the observations made in *this* experiment, write a statement of the relation of the apparent loss of weight of a body immersed in water to other weights of the body you have determined. Why does an immersed body appear to lose weight? Would this same relation exist for liquids other than water? Explain.

4. What relation did you find between the density and the specific gravity of the cylinder? Why does this relation exist? If the density were measured in slugs per cubic foot, would the relation be any different? Explain.

5. Assume the overflow can is filled and preparations are ready for catching the overflow water. A block that will float is now carefully placed in the can, and the overflow water is caught and measured. The experiment is then repeated with gasoline in the can. Discuss the relative volumes and masses of the two overflow liquids. (Note: Gasoline is less dense than water.)

6. One kilogram of iron and one kilogram of aluminum are submerged in water and their apparent weights are recorded. How do the apparent weights compare (qualitatively)? Explain.

7. One cubic centimeter of aluminum and one cubic centimeter of lead are each weighed in air and then in water. How do their apparent weight losses compare? Explain.

8. Suppose a vessel of water is balanced on a laboratory balance. Will the balance be disturbed if you put your finger into the water? Why? If in doubt, try it.

9. In general, do you think the density of a body depends on its temperature? Why ?

10. If a body floats in a liquid, how does the loss of weight (buoyant force) compare with the weight of the body? Explain.

11. What is the density of the irregular solid used in the experiment in grams per cubic centimeter? In the SI system?

12. Can you suggest a way to use Archimedes' principle to find the length of a tangled bundle of wire without undoing the tangle?

RECORD OF DATA AND RESULTS

Density of a Solid Cylinder

	Cylinder no. 1	Cylinder no. 2
Material		
Mass (grams)		
Length (cm)		
Diameter (cm)		
Volume (cm)		
Density (g/ cm^3)		
Specific gravity		

Displacement Data

<table>
<tr><td colspan="2"></td><td colspan="2">Cylinder
Steps 4, 5 & 6</td><td>Irregular Solid
Step 7</td></tr>
<tr><td colspan="2">Material</td><td colspan="2"></td><td></td></tr>
<tr><td colspan="2">Mass</td><td colspan="2"></td><td></td></tr>
<tr><td colspan="2">Mass of empty catch vessel</td><td colspan="2"></td><td></td></tr>
<tr><td colspan="2">Mass of vessel + overflow water</td><td colspan="2"></td><td></td></tr>
<tr><td colspan="2">Mass of overflow water</td><td colspan="2"></td><td></td></tr>
<tr><td colspan="2">Balance reading of solid in water</td><td colspan="2"></td><td></td></tr>
<tr><td colspan="2">Difference when suspended
in air and water</td><td colspan="2"></td><td></td></tr>
<tr><td>Density of cylinder</td><td>Step 2 =</td><td>Step 5 =</td><td colspan="2">% difference =</td></tr>
<tr><td>Sp. Gr. Of cylinder</td><td>Step 5 =</td><td>Step 7 =</td><td colspan="2">% difference =</td></tr>
<tr><td>Sp. Gr. of irreg. obj.</td><td>Step 2 & 4
Method</td><td>Step 7
Method</td><td colspan="2">% difference =</td></tr>
</table>

Specific Gravity of a Liquid by Specific Gravity Bottle (Step 9)

Mass of empty, dry pycnometer (bottle) ______________________g

Mass of pycnometer full of liquid x ______________________g

Mass of liquid x in pycnometer ______________________g

Mass of pycnometer full of water ______________________g

Mass of water in pycnometer ______________________g

Specific gravity of liquid x ________________________

Density of a Liquid by Direct Measurements of Mass and Volume (Step 10)

Balance reading with dry, empty graduated cylinder ________________________g

Balance reading with graduated cylinder plus alcohol ________________________g

Net mass of alcohol ________________________g

Volume of alcohol ______________________ml

Density of alcohol (g/ml) ___________________(g/ml)

Density of alcohol in SI units __________________(kg/m^3)

Chapter 4: Electricity

16. Electric Field Mapping

Introduction

Positive and negative charges produce an *Electric Field*, $\boldsymbol{E}$, at every point in space. The force experienced by a charge, q, in such an electric field is $\boldsymbol{F}=q\boldsymbol{E}$. Electric field lines begin and end on charges, since these are the sources of the electric field. We represent the electric field by vectors which point in the direction of the force on a positive test charge: away from other positive charges and towards negative charges. Electric Fields may be measured by looking at the acceleration of a charged particle in the field, but this can be difficult to do. It is easier in the lab to measure the electric *potential* or voltage.

Equipment

Shallow glass pan	Pads of graph or grid paper	4 electric leads
tap water (salt)	metal shapes DC power supply	voltmeter or multimeter

Theory

As a charge moves along an electric field line, work is done by the electrical force. The energy gained or lost by this charge moving in the field is a form of *potential energy*, and so associated with the electric field is an *electric potential*, V, which has units of Energy per charge or Joules per Coulomb, which we also call Volts. Since voltage is potential energy per unit charge, voltage increases when going from a negative charge towards a positive charge. (The kinetic energy of a positive charge would increase when going from a higher potential to a lower potential.) A surface along which the potential is constant is called an *Equipotential*. On a piece of paper, the equipotential is represented by a line on which the voltage is constant. These equipotential lines may be mapped by using a voltmeter to find the lines of constant voltage. (If the constant voltage extends over a volume of space, then the equipotential on the two-dimensional surface may be an area rather than simply a line.) At the end of this section is a discussion of gravitational equipotentials and forces.

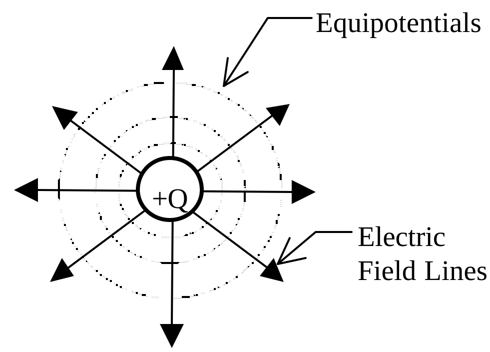


Figure 1: Equipotentials and Electric Field Lines around a spherical charge.

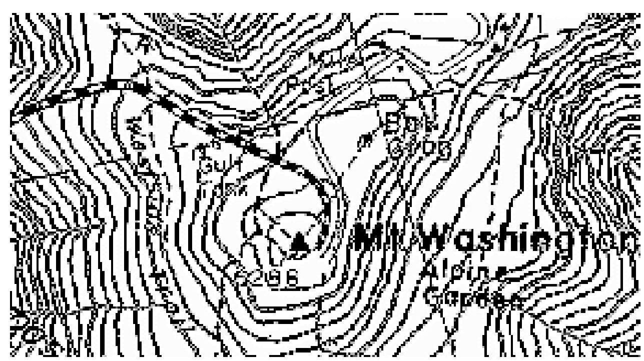


Figure 2: A topographical map. Lines of constant elevation (height) show how the gravitational potential energy is changing. (Some of the lines on this map are roads) The line of steepest descent is perpendicular to the equipotentials.

<u>**Gravitational Equipotentials and Forces**</u>

Since gravitational potential energy depends on height, lines of constant height would be gravitational equipotentials. A map of such lines is called a topographical map. Typically, a topographical map shows equally spaced lines of constant elevation. Where the lines are most closely spaced the elevation is changing most sharply, in other words the terrain is steep. The direction of the gravitational force (the component of gravity along the surface) would point down along the direction of steepest descent. The direction of the gravitational force is perpendicular to the gravitational equipotentials since the steepest downward slope is perpendicular to the lines of constant elevation.

Electric Field lines are also perpendicular to Equipotentials.

Since there is no work associated with moving in any direction along an equipotential, there can be no forces in any direction along the equipotential line or surface. Thus the vector ***E*** must have zero component along the equipotential surface: in other words it is perpendicular to the equipotential. Just as for the gravitational force, the electric force (and hence the electric field) points in the direction of the steepest change in electric potential.

In this lab you will look at the electric potential around different shapes of conductors. You will first use a voltmeter to map out lines of constant potential (the *equipotentials)*. After you have found and sketched the equipotential lines you will draw the electric field lines which are perpendicular to the equipotentials.

Procedure

Your conducting shapes will consist of metal shapes which you will place in a thin film of water. The conducting water (tap water generally has enough ions in it to conduct electricity, a *little* salt may be added if necessary) is needed in order to measure the *electric potential* (or *voltage*) at various points in space. If the negative terminal of the power supply is connected to one shape and also to the negative terminal of the voltmeter, then this shape will be at zero volts (Ground). The voltmeter will be able to measure the voltage difference between Ground and any point which the voltage probe touches.

Your instructor will show you how to connect the circuit. Do not turn on the power supply until you are sure it is connected properly.

Connect the circuit as shown by your instructor. Use two bars for your conducting shapes. Place a piece or two of graph paper under the glass tray to make a reference grid. ***Be sure and connect the shapes to the output labeled DC voltage.*** Have your multimeter set to measure DC voltage. Practice using the voltmeter to:

Measure the voltage produced by the power supply. Calibrate the variable knob on the supply by finding the setting corresponding to zero volts and

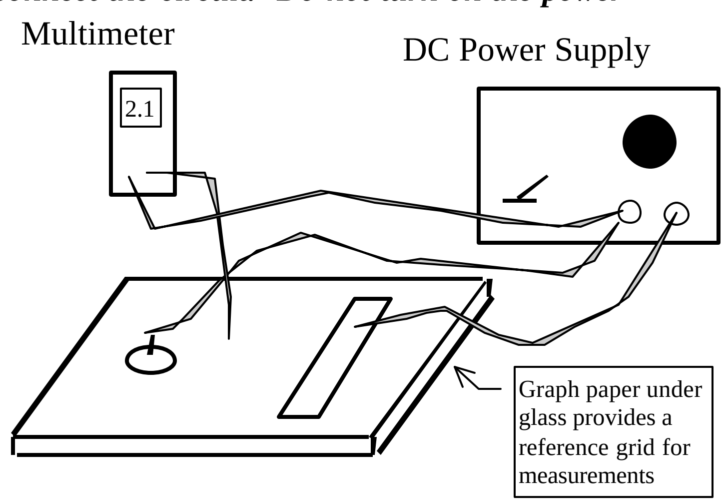


Figure 3: A DC power supply is connected to two conducting shapes, establishing a potential difference between them. A voltmeter is used to measure the potential (voltage) at various points between the two shapes.

to ten volts. Leave the power supply set at ten volts.

Now measure the voltage of each of your two shapes. One shape should be at zero volts and the second at ten volts.

<u>Verify that the entire conducting shape is at the same voltage</u>. Because electrons are free to move within a conductor, the entire conductor must be an equipotental (in a static situation).

Measure the voltage at several points between the two shapes. Does it vary between zero and ten volts? If not, you may need to check your circuit or add a little salt to the water to increase its conductivity.

If you are having difficulty measuring voltages or they do not seem to be correct, have your instructor check your setup. It is important to get the circuit correct before proceeding.

For your report:

Make a sketch of your circuit. Label the positive and negative terminals of both the power supply and the meter.

Each lab partner should use an entire sheet of paper to make an overhead view sketch of your two conducting shapes. Use the same type of graph paper as you placed under the glass so that your sketch can be full scale and accurately drawn. You may wish to draw axes on both your graph paper and the paper under the glass. Label the shape connected to the positive terminal "10 Volts" and the shape connected to the negative terminal "0 Volts".

Use the voltmeter to find a point between the shapes which is about 5 Volts. Draw an "X" on you sketch at the location of this point. You will now find an equipotential line by moving the voltage probe and tracing out a line which is at a constant voltage (in this case, 5 Volts).

For the following one (or more) lab partners should be in charge of sketching while another moves the probe and another keeps track of the voltmeter. Have one partner move the probe about 1 cm keeping while keeping the reading on the voltmeter at 5 Volts. On your sketch make an X at the second point and connect to two by a line to indicate an equipotential.

Continue to move the probe and record on your sketch an X for the points at which you check the voltage and the lines of constant voltage which connect them. The separation between X's may vary depending on how quickly the voltage changes. If the voltage is constant for a long straight line you might move the probe a longer distance before recording an X. If the voltage changes direction quickly you may need to record more X's in order to accurately sketch the equipotential lines.

Once you have sketched a line from the center between the two conductors out beyond the edges you may have to go back to the center and sketch the second half of the line in the opposite direction. If your shapes are symmetrical you may expect the field lines to be symmetrical as well. You may be able to make fewer X's: just check to see if the equipotential lines are indeed symmetric.

The line you have drawn is an equipotential with a value of 5 Volts. Label this line "5 Volts". For the two parallel bars make sure that you have investigated the areas out near (and beyond) the areas of the bars.

Lab partners should now switch roles so a different person measures the voltage or records the sketches. Now repeat the procedure to find *at least* two more equipotential lines: for voltages of 7.5 Volts and 2.5 Volts. Label them on your sketch.

For your report:

Each partner should have a sketch of the equipotential lines. Some of the sketches will be made during the voltage measurements. Additional copies can be made by tracing these sketches.

You now have 5 equipotential lines (counting the surfaces of the conductors). You can now draw the electric field lines which are perpendicular to the equipotentials using the following criteria:

The directions of the field lines go from high voltage to low voltage. (This is because high to low voltage would correspond to a decrease in potential energy (increase in kinetic energy) for a positively charged particle and so this is the direction of the electrical force on a positive charge).

The field lines should be perpendicular (90 degrees) to the equipotentials *The surfaces of the conductors themselves are equipotentials!* Start at one point on an equipotential line and draw a short line perpendicular to it. Then figure out how you would need to bend that line so that by the time it encounters the next equipotential line it is also perpendicular to that line.

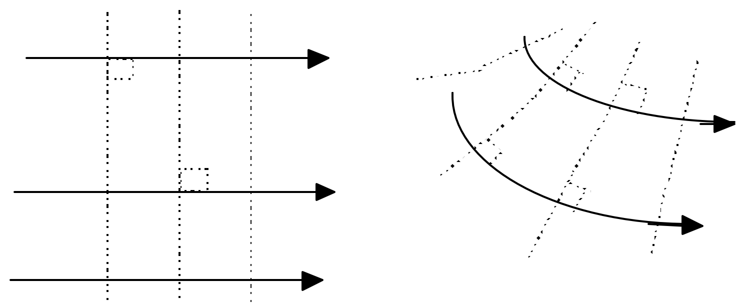


Figure 4: Equipotentials (dashed lines) and Electric Field Lines (solid). Electric field lines should be perpendicular to equipotentials at the points where they cross.

Draw a number of electric field lines which begin on the positively charged conductor (higher voltage) and end on the negatively charged conductor (lower voltage). You will want to sketch in a line lightly with a pencil and check with your instructor to make sure you understand the procedure.

For your report:

Draw the electric field lines on your sketch. Label the lines "E" and draw arrows along them to show the direction of an electric field. The arrows should go from higher voltage to lower voltage. At representative points draw a right angle symbol ( ) where the field lines meet the equipotentials.

Repeat for additional conducting shapes. ***Turn off the power supply while connecting and disconnecting the leads***. . You may try two round shapes or one round shape and one bar.

For your report:

Each partner should have a sketch of the equipotentials and field lines for the additional shapes.

Obtain a hollow metal ring and place it in the water to represent a hollow conductor. Place bar shaped electrodes on either side of the hollow ring. Investigate the change of potential at various points inside the hollow ring. Compare to the field without a ring.

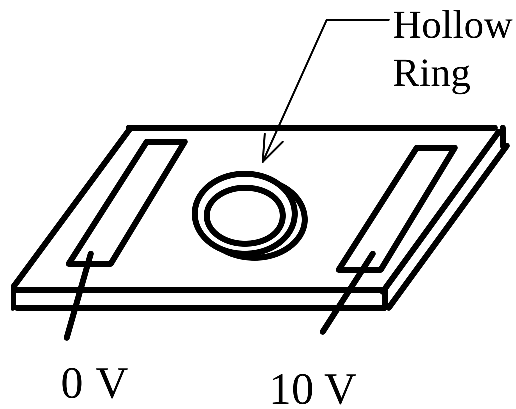


Figure 5: Hollow conducting ring placed between two electrodes.

For your report:

Make a sketch of your set-up and describe your results.

Your lab report should consist of an introductory cover page which includes the title, names of lab partners, a paragraph or two (*in your own words*) describing the ideas behind the experiment and the basic procedures. The data for this lab will consist of the sketches you have made. Summarize your conclusions by answering the following questions in complete sentences with an explanation of your reasoning.

Voltage is potential energy per unit charge. If you were a positive charge and you moved from a higher voltage (10 volts) to a lower voltage (0 volts) how would your kinetic energy change? What would the direction of the electrical force acting on you be?

How does your answer change if you are a negative charge?

What is the direction of the electric field in each case (is there any difference)?

How does the potential energy of a charge change as it moves *along* the surface of a conductor? What is the force on an electron along the surface of the conductor (in other words the component of force in the direction of the surface). What is the component of electric field along the surface of a conductor?

When the equipotential lines bend, what happens to the electric field lines?

Comment on the equipotential lines and the electric field between the two parallel bars. If the bars were infinitely long, what would you expect the electric field between them to look like?

Comment on the equipotentials and field lines for two spherical shapes or a sphere and a bar. How does the symmetry of the configuration show up in the equipotentials and fields?

17. Batteries 'n Bulbs: Voltage, Current and Resistance.

Introduction

A simple electric circuit can be made from a *voltage* source (batteries), wires through which *current* flows and a load, such as a light bulb, which provides *resistance*. In this lab you will use the multimeter to measure voltage and resistance and to begin to learn about simple DC circuits.

***Voltage**:* Batteries a source of "electromotive force" (EMF) for a circuit. Chemical reactions in the battery maintain a constant potential difference, that is a voltage, between the positive and negative terminals of the battery. If the battery is connected to a circuit with conducting wires then electrons will move from the negative terminal ($U = q\,V$ = lower potential energy for the negative electrons) to positive terminal ($U = qV$ = higher potential energy). The symbol for voltage is V and its units are Joules/Coulomb or Volts (V).

Current: Even though we now know it is the electrons which actually move, by convention current is defined as the direction of movement of positive charge. Thus the conventional positive current is said to flow from the higher voltage (positive terminal) to the lower voltage (negative terminal). The symbol for current is I and its units are Coulombs/Second or Amperes also known as Amps (A).

***Resistance**:* Resistance is a measure of how hard it is for current to move through a conductor. The symbol for resistance is R and its units are Ohms (Ω). Electrical devices which have a linear relationship between voltage and current obey "Ohm's Law":

$$V = I\,R \qquad \text{("Ohm's Law")}$$

Not all electrical devices are "ohmic". Circuit elements such as capacitors, transistors, etc. have a more complex relationship between voltage and current.

Equipment

Two batteries	Battery Holders	8-10 alligator clips	3 light bulbs (different types)
Light bulb holder	multimeter	probes for meter	

Note: light bulbs should be different types, at least one with significantly different resistance.

Procedure

For each part follow the suggested procedure then stop and write your observations, answer questions and summarize conclusions. <u>Your report should have a title page with the lab title, your name, date and time, lab partners and a brief introduction. Label each section of the report with a section heading.</u>

Make sure everyone in your group has a chance for hands on investigation.

Examine the batteries. They will have terminals labeled + and -. Determine which end is positive and which end is negative.

Set the multimeter as a voltmeter to measure DC Voltage (V). Make sure the leads are in the proper sockets and if there is a switch it is set to DC rather than AV voltage. Be careful to note whether the voltage reading is in V or mV (10^{-3} V). A voltmeter measures a voltage <u>difference</u> between to points. Measure the voltage difference between the positive and negative terminals of the batteries. If you switch your leads you should notice that the reading on the meter changes sign since the voltage difference changes sign.

Now you will investigate different configurations of batteries. Place the batteries in the holders and use double sided alligator clips to connect them as shown in Figure 6. For each configuration measure the voltage between the bottom of the batteries (the point labeled (A)) and the top of each battery (B) and (C). Make sure that the Common or Negative terminal of the voltmeter is connected to point (A). Note: since wires in our circuit are considered ideal conductors, if a wire is connecting two points in a circuit they should have the *same potential* (voltage). Verify this. To make your voltage measurements you can touch or clip your multimeter probes to the battery holders. Make sure you have good electrical contact.

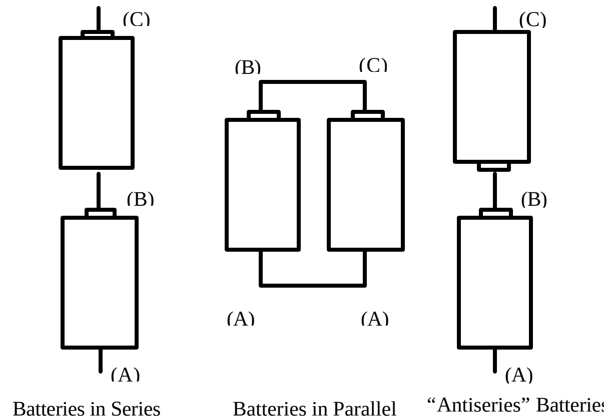


Figure 6: Battery configurations

For your report:

Make a sketch of each configuration *Make your sketches large and easy to read.* Label the points you use to measure voltages and record the voltage next to the sketch. (For example "Voltage between (A) and (B) = V_{AB} =).

Questions:

A chemical reaction in the battery tries to maintain a constant potential (voltage) difference between the negative and positive terminal of the battery. Do you observations support this? (Hint: for series and anti-series configurations the differences should be cumulative.)

Part 2: Anatomy of a light bulb

In this part you will investigate the anatomy of the light bulb and its socket. Your mission is to find out 1) why there are two electrical contacts on the bulb socket and 2) which parts of the light bulb contact which parts of the socket.

Investigate the bulb socket. Remove one of the bulbs. Determine which electrical connectors are connected to different parts of the bulb socket (There are nails holding the socket holder to the board: these are not part of the bulb socket! The inside of the bulb holder is an important however!).

Now study the light bulb: how is it constructed? Which parts of the bulb come in contact with which parts of the socket?

Connect a battery to the bulb by connecting an alligator clip from each end of the battery to each electrical contact of the socket. The bulb should light up at least a little. What happens when you disconnect on clip? Does it matter which clip you disconnect? What happens when you partly unscrew the bulb?

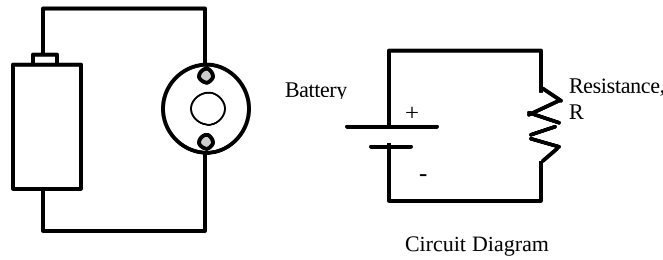


Figure 7: Circuit with a single battery and bulb. The circuit diagram symbols for a battery and a resistor are shown. Wires are represented as lines.

For your report:

Make a sketch of the light bulb (a side view showing the filament and how it is connected inside the bulb). When the light bulb is connected to the battery, the potential difference in the battery should cause positive current to flow through the wires and bulb from the positive terminal to the negative terminal. (It is actually electrons moving in the opposite direction). Draw the battery connected to the bulb and show on your sketch how current flows through the bulb. Label the current with an "I".

Question:

What is needed in order for the lightbulb to light up?

Now you will study light bulbs in parallel and in series. The circuit diagrams are shown in Figure 8. Note that the shape of the circuit diagram does not have to correspond to the physical shape of the circuit. What is important is 1) where the current flows and 2) which points are connected by wires (and therefore at the same voltage).

Part 3: Bulbs in a Parallel Circuit

Construct the top circuit.

If your bulbs have different brightnesses, move them around to determine whether the brightness depends on the position in the circuit. If it does not, then they are different brightnesses because the are different types of bulbs.

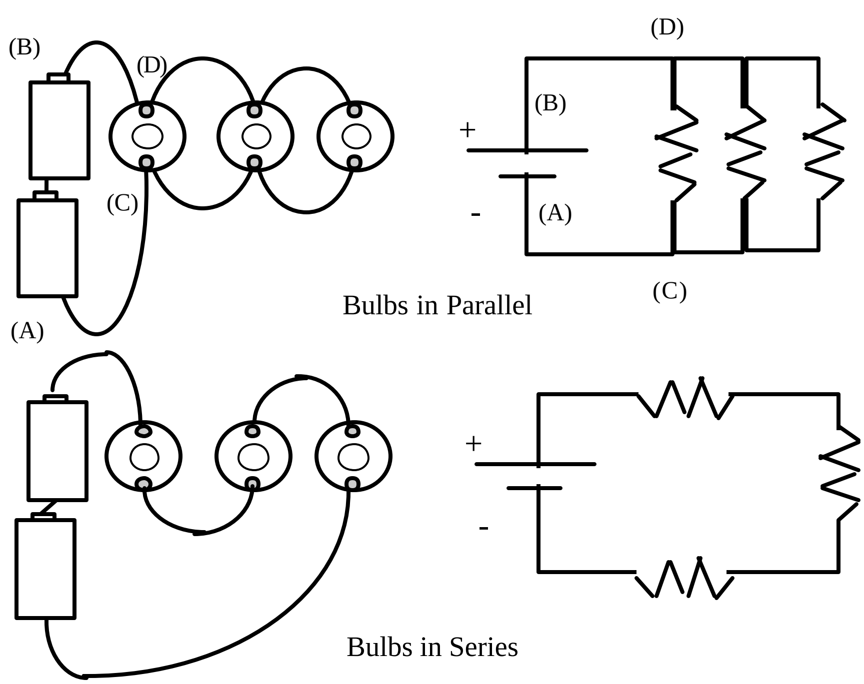


Figure 8: Light bulb Circuit One (top) and Circuit Two (bottom). Corresponding circuit diagrams on the right.

For your report:

Make a (LARGE) sketch of the top circuit and label it "Parallel Circuit". Draw the circuit diagram next to it.

Label the points on each side of the battery and on each side of the bulbs (A), (B), (C), etc. Label the corresponding points on your circuit diagram (as shown in the top circuit in Figure 8). Set your multimeter up as a DC voltmeter and record the voltage differences across 1) the combination of the two batteries and 2) each of the bulbs. Record these values below your sketch. For example, the voltage between (A) and (B) should be measured with the negative probe at (A) and recorded as V_{AB} . Answer the following for this circuit:

Questions:

What happens when you unscrew one bulb? Two bulbs? Does the brightness of the remaining bulb(s) change?

How does the voltage across each bulb compare? How does it compare to the voltage across the battery.

Set the multimeter as an ohmmeter to measure Resistance (Ohms or Ω). Make sure the leads are in the proper sockets. The resistance may be reported in MΩ (10^6 Ω), k Ω (10^3 Ω) or Ω. Resistance is a measure of how hard it is for current to flow. A good conductor will have (nearly) zero resistance, a good insulator will have very large resistance. Touch the two multimeter probes together to measure the resistance of the test leads. Hold them apart to measure the resistance of the air. (If the resistance is larger than the meter is capable of reading it should indicate this). Do your readings make sense?

Disconnect all of the alligator clips. The alligator clips will remain disconnected for this section. Change your multimeter to an Ohmmeter and measure the resistance across the first bulb. The value should be very small (an Ohm or a few Ohms, if you read kiloOhms your meter may be defective) Record this value as "R_1". Repeat for the second and third bulb and record "R_2" and "R_3". Label the resistors on your circuit diagram with the appropriate labels and values.

Now connect two alligator clips between R_1 and R_2 so they are in parallel again. Measure the Resistances across each bulb. You should get the same value for each. This is the effective resistance of the combination. Under the circuit diagram record this value as R_{1+2}. Connect up the third bulb in parallel (as in your original circuit) and measure the effective resistance across all three bulbs. Record this value under your diagram as R_{1+2+3}..

For your report:

You have measured the values for R_1, R_2 and R_3 and the effective values R_{1+2} and R_{1+2+3}. These are you experimental values. For a circuit in parallel the effective resistance of should be

$$\frac{1}{R_{1+2}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \text{and} \qquad \frac{1}{R_{1+2+3}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

These are the theoretical values for resistors in parallel. Use the measured values for R_1, R_2, and R_3 to calculate the expected theoretical value for R_{1+2} and R_{1+2+3} and record these labeled "theoretical values.

Questions:

Is there a correlation between the Resistances you measure and the brightness of the bulbs? (If you can't remember connect the battery up again) If Ohms Law predicts that V=IR how do you make sense of this? (Remember that you previously determined the voltage across each lightbulb).

How do the theoretical values for expected resistance compare to the experimental value for R? (The agreement may not be exact, notice however if the effective resistance is larger or smaller than any of the individual resistance.)

Does resistance get larger or smaller when you combine resistors (or bulbs) in parallel?

Construct the bottom circuit. (Note: depending on your bulbs some may light up very dimly here.)

For your report:

Make a (LARGE) sketch of the bottom circuit and label it “Series Circuit”. Draw the circuit diagram next to it. Label the points on each side of the bulbs (A), (B), (C), etc. Label the corresponding points on your circuit diagram. Set your multimeter up as a DC voltmeter and record the voltage differences across 1) the combination of the two batteries and 2) each of the bulbs. Be sure to measure the voltage going the same direction around the circuit so that the measured voltages are all the same sign. Record these values below your sketch. Answer the following for this circuit:

Questions:

What happens when you unscrew one bulb? Any bulb?

How does the voltage across each bulb compare?

What is the sum of the voltages across all three bulbs? How does it compare to the battery voltage.

Do you think the same current is flowing through each bulb? Explain why or why not. (Do not rely on the brightness of the bulbs, consider the circuit diagram).

Disconnect all of the alligator clips. The batteries will remain disconnected for this section. Make sure that you remember which bulb is which and label the resistors on your circuit diagram with the appropriate labels and values for R_1, R_2, and R_3.

Now connect the alligator clips between R_1 and R_2 so that they are in series. Measure the across BOTH bulbs. Under the circuit diagram record this value as R_{1+2}.

Connect up the third bulb and measure the effective resistance across all three bulbs. Record this value under your diagram as R_{1+2+3}..

For your report:

Record the three resistances, R_1 , R_2 and R_3 and the two effective resistances R_{1+2} and R_{1+2+3}. These are your experimental values. For a circuit in series the effective resistance of should be

$$R_{1+2} = R_1 + R_2 \qquad \text{and} \qquad R_{1+2+3} = R_1 + R_2 + R_3$$

These are the theoretical values. Calculate the expected theoretical values and record them labeled as “theoretical values”.

Questions:

How do the theoretical values for expected resistance compare to the experimental values. Again they may not agree exactly but notice if the effective values are larger or smaller than the individual values.

Which circuit makes for brighter bulbs? The circuit in parallel or the circuit in series? You have measured the voltages for each, can Ohm’s Law explain the brightness?

Does resistance get larger or smaller when you combine resistors (or bulbs) in series?

Different circuit elements are in parallel when *they have the same voltage (potential difference) across them.*

Different circuit elements are in series when *they have the same current running through them.*

Write a paragraph summarizing your conclusions and comparing the two types of circuits (you may have recorded some of your observations earlier but you should summarize them at the end of your report as well).

18. Ohm's Law and Resistivity

Introduction

In this lab you will investigate simple DC (direct or constant current) circuits made with a DC power supply and resistors. You will be able to measure voltage, current, and resistance for different components in your circuits.

Equipment

Power Supply with Voltage and Current Display (HP)	Rheostat (85 or 115 Ω, 2+ Amps)	Multimeter, leads
Resistance Demonstrator	Alligator and banana plug leads	

Theory

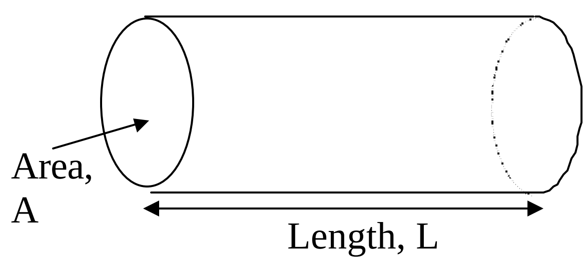
Ohm's Law $V = I\ R$

Resistance is a measure of how difficult it is for current to flow through a circuit element (a wire, a resistor, etc.) "Ohmic" devices have a linear relationship between voltage and current.

Power $P = I\ V$ which for an Ohmic device yeilds $P=IV= I^2R = V^2/R$

When energy is dissipated in a resistive load the rate of at which energy is dissipated is P=IV (Power in units of Watts=Joules/second) is Current (I=Charge flow/scconds in units of Ampere=Coulomb/scc) times Energy per charge (V=Voltage=Joules/Coulomb). For an Ohmic device, the voltage drop across the resistance is V=IR, so we get the result $P=I^2R$. If you double the current through a given resistance, then you will quadruple the rate at which energy is dissipated.

Resistivity $$R = r\frac{L}{A}$$



The resistivity, r, is a characteristic of a given material which describes how hard it is for current to go through that material. Conductors have very low resistivity, insulators have very high resistivity. Resistance (R) is the property of a particular piece of wire, carbon, etc. Resistance is determined by the resistivity (what kind of material), the length of the material (*L*) and its cross sectional area, (*A.*)

Material	Resistivity (Ω-m)		Material	Resistivity (Ω-m)
Copper	1.7×10^{-8}		Gold	2.44×10^{-8}
Silver	1.59×10^{-8}		Glass	10^{10} to 10^{14}
Tungsten	5.6×10^{-8}		Silicon	640

SAFETY NOTES

Turn off the current - (by unplugging one lead from the power supply - and note that the current reading becomes ZERO every time you re-arrange the circuit. The circuit should be safe to touch, but this is always a good safety precaution that will protect both you and the circuit elements!

Procedure

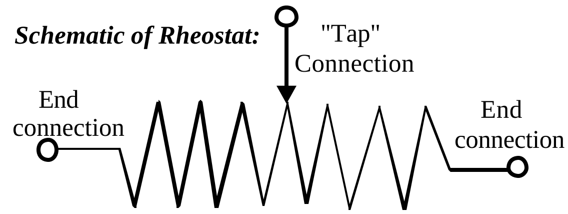
With no leads connected to the power supply:

Set up the Power Supply so it is on the 2 Amp setting (there is a button to select 2A or 3A).

Press in AND HOLD the "CC Set" button - the current limit will be displayed. Set the current limit to about 1 Amp. Do this by adjusting the Current knob.

A Rheostat is a variable resistor made of wire. You should have either an approximately 85 or 115 Ω Rheostat that is rated for about 2 Amps of current. The Rheostat is made of a long wire wound in a cylindrical shape. There are connections for each end of the wire - between them is a constant resistance. There is also a "wheel" that runs back and forth along the Rheostat. There is a third connection (or one at both ends of the bar running above the wound wire) that connects to this "tap". This allows you to connect to the Rheostat any point along the wire. The symbol for a Rheostat represents a resistor with a connection at each end and also a variable "tap" connection.

Schematic of Rheostat:



Use the Ohmmeter of your multimeter to make sure you understand the connections on the Rheostat.

Record the total resistance of the Rheostat as measured by you multimeter.

Relationship between current and resistance.

Turn the voltage of the power supply down to ZERO.

Connect the power supply across the entire resistance of the Rheostat. If it is an 85 Ω Rheostat there should be 85 Ω of resistance across the power supply.

Starting with ZERO Volts, make a record of Voltage (V) and Current (I) as you turn up the voltage on the power supply. You should record about 8 data points as you turn the voltage up from ZERO to the MAXUMUM (about 15 Volts on the 2 Amp setting.)

For each data point calculate the Resistive load (R) by using Ohm's Law V=IR i.e., R=V/I

MAKE A GRAPH of Voltage (V) versus Current(I). Use the curve fitting procedure (with Kalaidagraph for example) to fit the graph.

REMOVE ONE LEAD FROM THE POWER SUPPLY SO THE CURRENT READING IS ZERO. Leave the power supply on its highest voltage setting.

Move the center tap of the Rheostat so that it is about 20% of the distance from one end.

Measure and record the resistance between the end and the tap - REMEMBER you only get a true value of the resistance when all other circuit elements are disconnected.

Connect the power supply leads between one end of the Rheostat and the center tap.

Reconnect both leads to the power supply - record voltage and resistance.

Calculate the resistance using Ohm's Law and compare to the multimeter reading.

REPEAT for 3 additional positions of the Rheostat. DISCONNECT THE POWER SUPPLY EACH TIME YOU ARE MOVING THE RHEOSTAT TAP.

Questions (Part A and B) - .

Look at the numerical values you have calculated for resistance. What is the approximate uncertainty in the resistance?

Are the discrepancies between the value you measured with the Ohmmeter and the values you calculated *random* (equally high and low) or *systematic* (consistently high or low). Discuss the discrepancies - what are possible reasons?

Look at your graph from Part A - what evidence does it show that the Rheostat is an *Ohmic* device.

The resistivity demonstrator consists of five coils of wire. Each are labeled with the material (Cu=Copper or NiAg=Nickel Silver). The wires are clad in copper colored insulation. There are leads that allow you to connect to the ends of the wire. Each is labeled with a length (in meters) and a diameter.

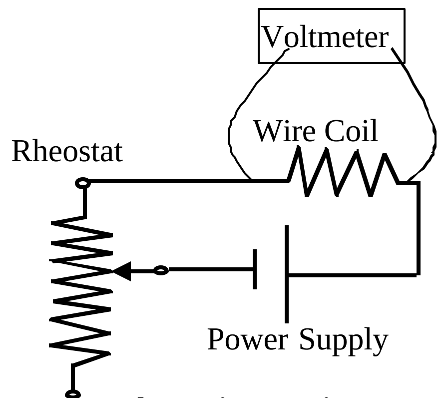
THE DIAMETER OF THE WIRES IS IN INCHES!!!!!! (2.54 cm equals 1in).

REMOVE ONE LEAD FROM THE POWER SUPPLY SO THE CURRENT READING IS ZERO.

Adjust the Rheostat so that its resistance is about 75Ω. Verify with the Ohmeter.

Connect the Rheostat IN SERIES with one coil of the Resistivity Demonstrator - it will take ONE additional alligator clip lead to do this. The new alligator clip goes between the Rheostat and the wire coil - you can connect to ONE of the wire coils where they are soldered to the terminals. The circuit is in SERIES if the same current that goes through the Rheostat also goes through the coil. NOTE - nothing is connected to the second end terminal of the Rheostat - there is no current in that part of the circuit.

Connect the DC Voltmeter (you multimeter) across the wire coil - you can connect to the terminals of the SAME coil you have connected to the Rheostat. With no current the voltage should read zero - if it does not record the milivolts of offset from zero.



Schematic Drawing

Once the circuit is built and double checked (YOU DO NOT WANT CURRENT TO GO ONLY THROUGH THE WIRE COIL IT MUST GO THROUGH THE RHEOSTAT AS WELL.) Reconnect the disconnected lead from your power supply.

Record both the voltage across the coil (from the multimeter *NOT* the power supply) and current (from the power supply display, since there is only one current in the resistor.)- write down the numbers as soon as you connect the lead. You may notice the current changing if you wait too long. Calculate the resistance of the wire.

DISCONNECT THE POWER SUPPLY - VERIFY THAT THE CURRENT IS ZERO.

Connect to a different coil of wire and repeat for all five coils.

DISCONNECT THE POWER SUPPLY - VERIFY THAT THE CURRENT IS ZERO.

Change the Rheostat setting so it is about 30Ω - verify with the Ohmmeter - measure Voltage and Current again for each coil of wire. Calculate Resistance of the coils again.

From Resistance (R) and knowing the length (L) and cross sectional area (A=pr^2) you can calculate the resistivity, r . NOTE THAT THE WIRE DIAMETER (not radius) IS GIVEN IN INCHES (not meters). Calculate the resistivity in units of Ω-m for each coil and for the two current settings.

(Continued next page)

As temperature increases the resistivity of metals generally increases. Passing current through a wire dissipates energy as heat. The rate at which energy is dissipated is the Power (Energy per second) . For Ohmic devices this is given by the equation $P = I\,V = I^2R = V^2/R$

Using the approximately 30Ω resistance to control the current in the circuit, go back and determine how the current in each coil changes as a function of time. Record the Current and the Voltage across the coil ***immediately*** after the power supply is connected to the coil (IN THE SAME CIRCUIT AS ABOVE - THE RHEOSTAT STILL LIMITS THE TOTAL CURRENT) and then leave the power supply connected for about 2 minutes. Record the Voltage across the coil after 2 minutes and calculate the percent change. Note whether it is an increase or decrease.

For each wire calculate the power dissipated WHEN THE CURRENT IS FIRST TURNED ON.

Questions (Part A and B) - .(THIS IS REPEATED FROM ABOVE FOR YOUR CONVENIENCE!)

Look at the numerical values you have calculated for resistance. What is the approximate uncertainty in the resistance?

Are the discrepancies between the value you measured with the Ohmmeter and the values you calculated *random* (equally high and low) or *systematic* (consistently high or low). Discuss the discrepancies - what are possible reasons?

Look at your graph from Part A - what evidence does it show that the Rheostat is an *Ohmic* device.

Questions (Part C and D) - .

Compare the resistance for Copper wires of the SAME radius but DIFFERENT lengths. CALCULATE the ratio of lengths and compare to the ratio of resistances.

Compare the resistances for Copper wires the SAME length but DIFFERENT radii - CALCULATE the ratio of cross-sectional areas and compare to the ratio of resistances.

Are the above results consistent with the theoretical relationship between Resistance and Resistivity?

What is discrepancy between your experimentally determined value for the resistivity of copper and the theoretical value (see table on the first page)? Is the discrepancy RANDOM or SYSTEMATIC?

The Ni-Ag wire does not have a theoretical value - WITHIN THE EXPERIMENTAL UNCERTAINTY can you say whether it's resistivity differs from copper? Which is the better conductor (if you can say).

Which wires showed the most changes in resistance when the power supply stayed on - EXPLAIN why these wires showed more change.

Total Resistance of Rheostat = ___________________________

Voltage V	Current, I	R from Ohms Law		% Difference from multimeter

Approx position	Voltage	Current	R from Ohms Law	R from multimeter	% Difference

SAMPLE CALCULATIONS:

Record Coil Characteristics:

Coil #	Material	Length, L, meters	Diameter, Inches	Radius, R, meters	Cross Sectional Area, A, m^2
1					
2					
3					
4					
5					

Calculations for Area:

Trial One: Current =

Coil # Material	Voltage across coil	Current Through coil	Calculated Resistance, R	Calculated Resistivity $r_{Exp.}$	Theoretical. Resistivity, $r_{Theo.}$	% Diff.

Trial Two: Current =

Coil # Material	Voltage across coil	Current Through coil	Calculated Resistance, R	Calculated Resistivity $r_{Exp.}$	Theoretical. Resistivity, $r_{Theo.}$	% Diff.

Coil #/ Material	Current (t=0)	V_{Coil} (t=0)	V_{Coil} (2 minutes)	% Change	Power at time t=0

Show sample calculations on separate page.

19. Resistances in parallel and in series.

Introduction

In this lab you will investigate simple DC (direct or constant current) circuits made with a DC power supply and resistors. You will be able to measure voltage, current, and resistance for different components in your circuits.

Equipment

Protoboard with voltage supply	Resistors	Wires	Multimeter and analog ammeter, alligator clip probes

Have lots of spare fuses on hand for the ammeter!!!!

Theory

To understand the relationship between current (I), voltage (V), and resistance (R) in a circuit, we can use a few simple relationships:

Ohm's Law $V = I\ R$

Resistance is a measure of how difficult it is for current to flow through a circuit element (a wire, a resistor, etc.) "Ohmic" devices have a linear relationship between voltage and current.

Kirchhoff's Rules

The sum of currents going into any node in a circuit must equal the sum of currents leaving the node.

The sum of voltage changes around any loop in a circuit is zero

"Kirchhoff's Rules" arise from the principles of conservation of charge and conservation of energy. The first rule simply says that since current is moving charge the number of Coulombs per second (Amperes) going into any junction of a circuit must be the same as the total current leaving that junction. For the node shown in the top frame of Figure 9 this can be written as

$$I_A = I_B + I_C$$

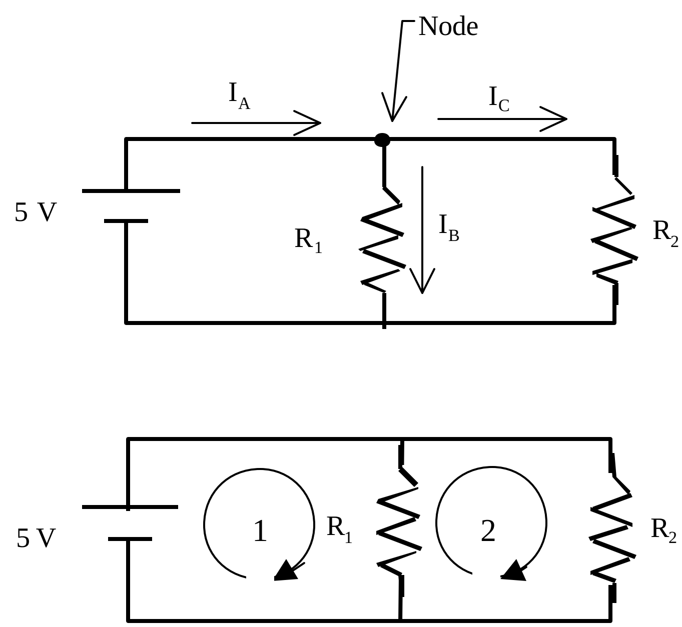


Figure 9: The current going into a node must equal the current leaving a node (Top). The voltage change going around any closed loop must be zero (bottom).

The second rule says that the change in potential energy per unit charge (that is the change in voltage) must be zero when you make a complete loop around any path in a circuit. The bottom frame of Figure 9 shows a circuit with two loops. Using the currents defined in the top frame, the changes in voltage around the loops can be written

$$5\ V + V_{R1} = 5\ V - I_B R_1 = 0 \qquad \text{(loop 1)}$$

$$V_{R1} + V_{R2} = +I_B R_1 - I_C R_2 = 0 \qquad \text{(loop 2)}$$

Here we also use Ohm's Law which says that the voltage across a resistor should decrease by V=IR in the direction of the current. Notice that we follow both loops in a single direction. For loop 2, we go around the loop in the direction opposite to I_B but in the same direction as I_C, and so the voltage increases by I_BR_1 but decreases by I_cR_2. The three equations determined from Kirchhoff's Rules provide enough information to allow us to solve the three currents I_A , I_B and I_C.

Resistors in series have *the same current going through them*. Resistance increases since the current must pass through all the resistors. The effective resistance of several resistors in series is given by

$$R_{effective} = R_1 + R_2 + R_3 + \ldots \qquad \text{(in series)}$$

Resistors in parallel have *the same voltage across each of them*. The circuit in Figure 1 shows two resistors in parallel. Because the current can split up among the different paths, the effective resistance decreases. The effective resistance of several resistors in parallel is given by:

$$\frac{1}{R_{effective}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \qquad \text{(in parallel)}$$

***Voltage**:* When using a voltmeter to measure voltage, the probes are attached across to points in the circuit and the meter measures the voltage *difference* ΔV between those two points. The ideal voltmeter has a very high resistance so that it draws very little current from the circuit.

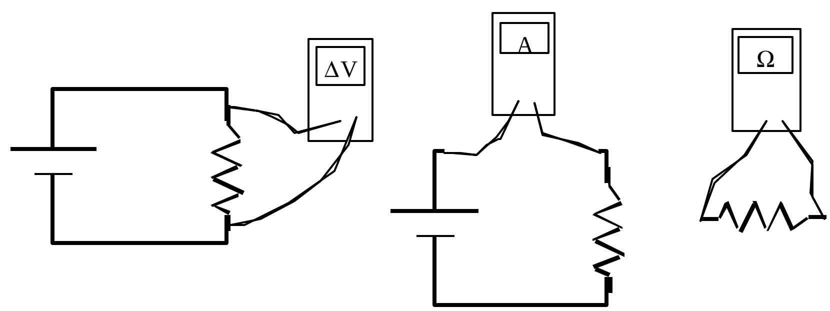


Figure 10: A voltmeter measures the voltage difference across two points in the circuit. An ammeter must be inserted into the circuit in order to measure current. When an ohmmeter is used to measure resistance, the circuit element must be disconnected from the rest of the circuit.

***Current**:* An ammeter is used to measure current in units of Amperes (or Amps). The ammeter must be inserted *in* the circuit so that the meter measures the current passing through it. The ideal ammeter has low resistance so that it doesn't affect the voltage in the circuit.

CAUTION: It is possible to damage the ammeter by passing too much current through it. Make sure it is properly connected (in series with a resistor as shown in Figure 10***). If too much current goes through the ammeter it will blow a fuse!***

***Resistance**:* An Ohmmeter is used to measure resistance in units of Ohms (Ω). A circuit element must be removed from the circuit in order to measure its resistance, otherwise you are not measuring the resistance of that element but rather the effective resistance of everything it is connected to.

Resistors are often color coded with their values. There are usually four bands of color. The first two bands represent two digits and the third band represents additional powers of ten. The fourth band is either silver or gold, indicating the value is good to 10% or to 5%. The numbers represented are:

Color	Black	Brown	Red	Orange	Yellow	Green	Blue	Violet	Gray	White
Value	0	1	2	3	4	5	6	7	8	9

A resistor with bands of Brown, Blue, Red and Gold would signify the three digits 162 and have a value of $16 \times 10^2 = 1600$ Ohms. Practice using your Ohmmeter to until you understand color code on several resistors.

Procedure

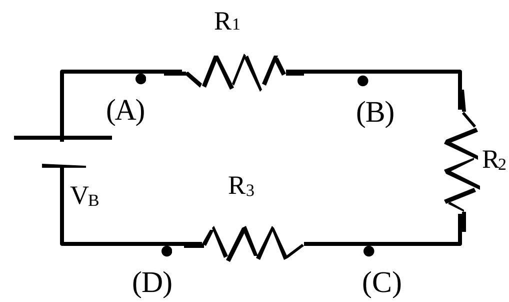


Figure 11: Series Circuit

Choose three resistors with different values. For a 5 Volts power supply, in the hundreds of Ohms will result in currents of 10 to 100 mA. Choose three resistors of different values between 100 and 999 Ohms.

Use your Ohmeter to measure the three resistances and record them in a data table.

Construct the series circuit shown.

Measure the voltages across each resistor. The current should flow from the positive terminal of the power supply to the negative and the voltages should drop across each resistor in the direction of the current. If you are going around the loop in a clockwise direction, make sure your voltmeter has its negative terminal the "same" side of each resistor. (For example across R_1 point (A) should be the negative terminal, across R_2 it should be point (B), across R_3 point (C) should be the negative terminal and across the battery point (D) should be negative. Record these values as V_{AB}, V_{BC}, etc.

Now you will use your multimeter as an ammeter to measure current. ***The meter must be inserted into the circuit to measure the current***. **BE SURE NOT TO BLOW THE FUSE ON YOUR METER BY USING IT INCORRECTLY** -- do not use it the same way as you would a voltmeter. To measure the current at point (A), for example, you would unplug the wire from the resistor and connect one side of the ammeter to the end of the wire. The other lead of the ammeter then is connected to the end of the resistor where the wire used to be-- the current you wish to measure goes through the ammeter but is still limited by also having to go through the resistor. Refer again to Figure 10.

Measure the currents at points A, B, C and D. BE SURE TO CHANGE THE SCALE ON YOUR AMMETER TO MATCH YOUR MEASUREMENT. If your measurement is too small for your multimeter you may need to use an analog ammeter. Record as I_A, I_B, etc.

Use Ohm's Law to predict the voltage drop through each resistor and compare to the measured values. The circuit should obey Ohm's Law to within about 10 percent. ($V_B = I R_{effective}$). If this is not the case the circuit may not be constructed properly, your meter may be inaccurate or you may not be using it properly. Stop and seek help if this is the case.

Calculate the effective resistance for all three resistors (see above) and compare to value determined from Ohm's Law: $V_{Batt}=I_{Batt}R_{effective}$.

Apply Kirchhoff's Voltage Rule by writing an equation for the voltages around the single loop in this circuit. Write it first with symbols, then use your numbers to check this rule.

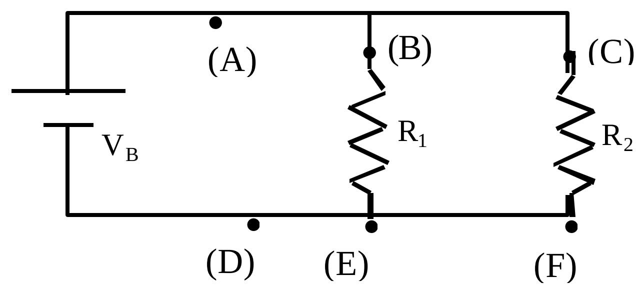


Figure 12: Parallel Circuit

Choose two resistors which have similar, but not identical, values. Measure the values with an Ohmeter and record them.

Construct the parallel circuit shown.

Measure the voltages across the power supply and the resistors.

Insert your ammeter within the circuit at points A, B, C and D and record the currents.

Do the resistors each obey Ohm's Law?

Calculate the effective resistance of the two resistors in *parallel* and determine whether the voltage across the battery is equal to the current from the battery times the effective resistance.

There are two loops in this circuit. Going around each loop clockwise, write Kirchhoff's voltage loop equations, first with symbols, then your numbers.

There are two nodes in this circuit. Draw pictures of the nodes and show the currents coming in and out. (Label them I_A, I_B, I_C or I_D) Write the equations for Kirchhoff's Rules as it applies to these nodes, first with symbols, then with your numbers.

Your report should include a title page with summary, sketches and data tables, equations. The final summary should address how well Ohm's Law or Kirchoff's rules hold and compare experimental values to theoretical predictions. (Comparisons should include typical percentage error). Discuss possible sources of discrepancy.

Deriving the effective resistances: (optional)

For the series circuit, use Kirchhoff's equation for the voltages then for each voltage substitute the equation $V_B = I R_{effective}$ to re-derive the expression for the effective resistance of resistors in series.

For the parallel circuit, use Kirchhoff's current equation for the top node on this circuit and then for each current substitute the relationships $I = V / R$ to re-derive the expression for the effective resistance of parallel resistors.

Series Circuit: Sketch the circuit and label resistors and points along the circuit where measurements where made. Record data below and label each number to correspond to your sketch (For example V_{AB} = 1.56V or I_B = .046 Amps.)

Record Data Here, then transcribe below for your calculations:

Battery:	V_{batt}=	R_1=	R_2=	R_3=
Voltages Measured:	V_{AB}=	V_{BC}=	V_{CD}=	
Currents Measured:	I_A=	I_B=	I_C=	I_D=

Data (insert symbol and value from above) and calculations:

	Resistor	Current for this Resistor	Voltage from Ohm's Law	Measured Voltage	% Difference
Resistors:	R_1=	I_A=			
	R_2=				
	R_3=				

Effective Resistance	R_{eff}= V_{batt}/I_{Batt}	
Theoretical Value	R_{eff}	
% Difference		

Kirchhoff's Rules for voltages around the series circuit (write out using symbols, then show the calculations using your numbers):

19. Resistances in parallel and in series.

<u>Parallel Circuit:</u> Sketch the circuit and label resistors and points along the circuit where measurements where made. Record data below and label each number to correspond to your sketch (For example V_{AB} = 1.56V or I_B = .046 Amps.)

Record Data Here, then transcribe below for calculations:

Battery:	V_{Batt}	$R_1=$	$R_2=$	
Voltages Measured:	$V_{BE}=$	$V_{CF}=$		
Currents Measured:	$I_A=$	$I_B=$	$I_C=$	$I_D=$

Data (insert symbol and value from above) and calculations:

	Resistor	Current for this Resistor	Voltage from Ohm's Law	Measured Voltage	% Difference
Resistors:	$R_1=$	$I_B=$			
	$R_2=$				

Effective Resistance	$R_{eff}=V_{batt}/I_{Batt}$	
Theoretical Value	R_{eff}	
% Difference		

Wite the following using symbols, then show the calculations using your numbers.

Kirchhoff's Rules for Voltages around the two loops:

Loop 1: Loop 2:

Kirchhoff's Rules for Currents going into and out of a node:

19. Resistances in parallel and in series.

20. Multiloop Circuits: Kirchhoff's Rules.

Introduction

In this lab you will practice apply Kirchhoff's Rules to circuits containing more than one power supply and several possible current paths. Kirchhoff's Rules result when we apply conservation of energy and conservation of charge to a circuit. The resulting equations allow us to solve for the currents in various branches of a circuit.

Equipment

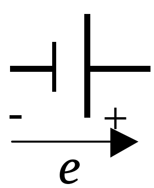
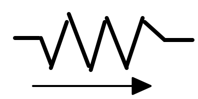
Protoboard	Resistors (~100-1000k)	Multimeter	Colored
Floating power supply	Wires, Aligator Clips		Pencils

Theory

Kirchhoff's Rules and **Ohm's Law** have already been discussed in previous labs.

Voltage Rule: The sum of voltage around a loop is zero $\left(\sum V = 0\right)$. This is a statement of conservation of energy since changes in voltage (potential energy per charge) should add to zero around any closed loop in the circuit. Once you get back to your starting point the total change in voltage should add up to zero.

When Kirchoff's Rules are applied in a circuit, the signs of the voltage changes are particularly important. When the voltage changes are traced through a battery or power supply in the direction from minus to plus the potential increases. The potential supplied by a power source is called the EMF (electromotive force), ε. When a circuit is traced through a resistor *in the direction of the current* the potential decreases. According to **Ohm's Law** this voltage change is $V_R = I R$.

ε *(EMF)* is positive going from - to + on a battery, negative going the other way 	Voltage (V=IR) is negative in the direction of current across a resistor, positive when you trace the circuit the other way. 

Current Rule: A node is any junction in the circuit where the current can split into more than one path. The sum of current into a node is equal to the sum of current out of the node. $\left(\sum I_{in} = \sum I_{out}\right)$. This is a statement of conservation of charge: the same amount of charge per second (current) must leave a junction in the wire as arrives. Charges are not created or destroyed in the circuit -- they simply circulate in the circuit gaining energy (from a power supply or battery) or losing energy (dissipated as heat in a resistor, for example).

Power is defined as the *rate* of Energy gained or dissipated. This rate is given by

$$\text{Power: } P = I \cdot V$$

The equation is easy to understand since the voltage, V, is potential energy per charge and the current, I, is the charge per second passing through the circuit. The units of power are Watts (Joules/sec).

A source of EMF (battery or power supply) *supplies* power at the rate $P = I \cdot e$ if the current passes through it in the direction from - to +. If current runs through a battery "backwards" from + to - then it *uses up* energy at the rate $P = -I \cdot e$. This can be equivalent to recharging the battery.

A resistor dissipates energy as heat at the rate of $P = -I \cdot V = -I^2 \cdot R$.

Procedure

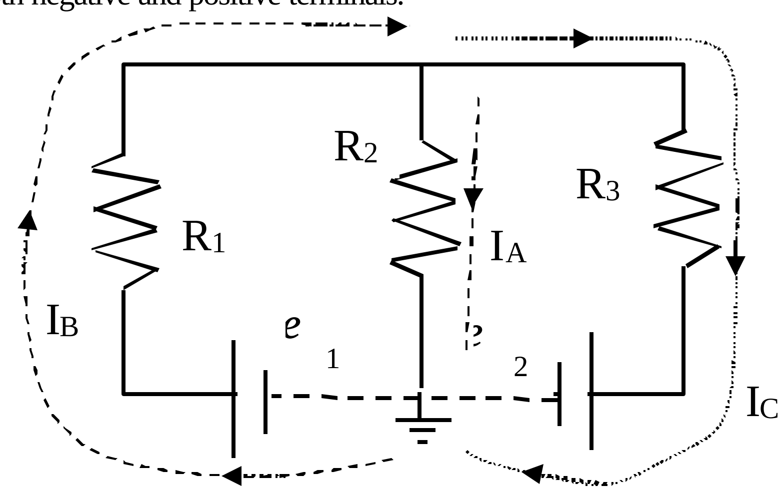
In this lab you will first apply Kirchhoff's rules to analyze a circuit and predict the currents in the circuit. After making your prediction you will construct the circuit on the protoboard and check your predictions. Things to keep in mind as you analyze the circuit are:

Make sure the signs of your voltages changes are correct (see above).

In order to solve an algebraic problem with N unknown quantities you need N independent equations. The number of equations you end up with must be the same as the number of unknown currents in your circuit.

The equations must be *independent*. That is you should not be able to combine any two equations to form a third: each new equation must contain new information about the circuit. You will need both voltage (loop) equations and current (node) equations.

This circuit may be constructed using two power supplies and three resistors. Choose resistors in the range of 100-1000 Ohms. If you use the protoboard supplies then the "negative" terminals do not exist -- the supplies are automatically referenced to ground. If you use external power supplies they may have both negative and positive terminals.



In the figure above the currents have already been drawn in and labeled. Directions for the currents were chosen arbitrarily. The current in the same in any path which does not have a split (junction) in it. Current does not change going through resistors or power supplies -- only at junctions.

The circuit above has two nodes (shown by small circles). There is only one independent current equation. For the bottom node the current going in is I_A+I_C and the current coming out is I_C. Thus

$$I_A+I_C = I_B \qquad \text{(Equation One)}$$

The top node would give an equivalent equation since I_B comes in and I_A and I_C go out.

There are two independent loops in this circuit. Tracing the left loop clockwise starting at the lower left we get the equation

$$-I_B R_1 - I_A R_2 + e_1 = 0 \qquad \text{(Equation Two)}$$

Tracing the right loop clockwise starting at the lower right we get

$$-e_2 + I_A R_2 - I_C R_3 = 0 \qquad \text{(Equation Three)}$$

The voltage differences are positive going through the battery minus to plus and negative going through the resistor in the direction of the current.

If we trace the circuit around the outside circle we would get the equation:

$$-e_2 + e_1 - I_B R_1 - I_C R_3 = 0$$

This is not independent however, since we could obtain it by adding Equations 2 and 3. There are three independent (one current and two voltage) equations and three unknown currents so it is possible to solve for the currents. From equations (2) and (3) both I_B and I_C could be written in terms of I_A. These expressions could be used in Equation (1) to write an equation with only I_A in it. Once this was found it would be possible to go back and find I_B and I_C to show that:

$$I_A = \frac{e_1 R_3 + e_2 R_1}{R_1R_2 + R_1R_3 + R_2R_3} \quad \text{and} \quad I_B = \frac{e_1 R_3 + e_1 R_2 - e_2 R_2}{R_1R_2 + R_1R_3 + R_2R_3} \quad \text{and} \quad I_C = \frac{e_1 R_2 - e_2 R_1 - e_2 R_2}{R_1R_2 + R_1R_3 + R_2R_3}$$

Construct this circuit using the protoboard and either internal or external power supplies. Use resistors from 100 to 1000 Ohms and the 5V and variable voltage power supplies. Make a sketch of your circuit and label all the power supplies and resistors with symbols *including subscripts.* Next to the sketch make a table showing the values you are using for power supplies and resistors. You can use colored pencils to color-code the different currents in your circuit. Indicate the labels for the currents as well.

Trace the circuit and measure the voltage differences across the power supplies and resistors. The power supplies on the protoboard are referenced to ground. *Be carefull of polarity -- your multimeter reads voltage differences between its two terminals. Put the leads in the same order as you are tracing your circuit.*

Record the voltage differences on your sketch. Indicate the direction you measured (with an arrow) and the sign of the voltage.

Determine whether the sum of voltage differences around any loop add to zero.

Below your sketch write the appropriate voltage equations -- first with symbols (including subscripts) then with the values you have measured.

Use the ammeter to measure all the currents in your circuit. Again, the meter can tell you the direction of the current by its sign.

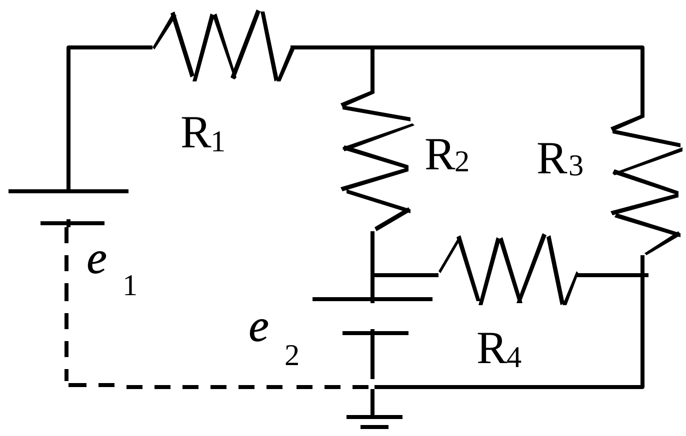
Record the currents on your sketch with arrows showing the direction.
Do the currents into each node add up to the currents coming out?

Calculate the power supplied or dissipated in the circuit.

Does power supplies equal power dissipated?

Compare measured currents to values calculated by solving Kirchhoff's equations. Show and label your work clearly.

RECOPY YOUR SKETCH AND DATA IF NECESSARY TO MAKE THEM NEAT AND READABLE!!!



Add another resistor. Make a sketch, labeling and color coding everything as before.

Choose directions and label and color-code the currents in this circuit

There will be one less independent current equation in this circuit than there are nodes. Write out the independent current equations.

How many independent loops are in this circuit? You have found all of them if the loops you have used have traced every wire. Write out the voltage equations.

You don't have to solve this one! Can you solve it (in principle) -- are there as many equations as you have unknown currents?

Measure the currents in this circuit. Pay careful attention to their directions. Record the data on your sketch.

Are the current node equations upheld? Write them out with both symbols and with numbers.

Are the voltage loop equations upheld? Use the currents you have measured to calculate the voltage across the resistors and check the loop equations. Again, be careful of directions!

Is energy conserved in this circuit (power supplied $\Rightarrow$ power dissipated). Show and explain your calculations.

RECOPY YOUR SKETCH AND DATA IF NECESSARY TO MAKE THEM NEAT AND READABLE!!!

21. Charging and discharging a capacitor.

Introduction

A capacitor is made up of two conductors that store positive and negative charge. When the capacitor is connected to a battery current will flow and the charge on the capacitor will increase until the voltage across the capacitor, determined by the relationship C=Q/V, is sufficient to stop current from flowing in the circuit. Figure 13 shows a circuit that can be used to charge and discharge a capacitor.

Equipment

Protoboard with power supply and switch	Capacitors (about 100µF)	Wires	ULI interface
	Resistors (values of about 50kΩ,100k, 200k)	Multimeter	Data Logger software

Theory

Before the switches are closed, there is no charge on the capacitor. When switch S_1 is closed, current will flow in the circuit as the capacitor is charged. According to Ohm's Law, the voltage across the resistor will be

$$V_R=IR$$

while the voltage across the capacitor will be given by

$$V_C=Q/C.$$

By Kirchhoff's Rule the voltage changes around the circuit must add to zero so

$$V_{batt} - V_R - V_C = V_{batt} - IR - Q/C = 0$$

When the capacitor charges the charge, Q, starts at zero and there is no voltage on the capacitor. This means the current initially flows at its maximum rate ($I_{Max}=V_{batt}/R$ when Q=0). However as the flowing current charges the capacitor, the voltage on the capacitor increases. This voltage opposes the flow of more charge and the current begins to decrease. The *rate* at which the capacitor charges slows as the current decreases -- as more and more charge builds up the current becomes smaller and smaller. The current *decreases exponentially* -- it asymptotically approaches zero for longer and longer times. Similarly the charge *increases* exponentially -- it keeps growing but at a slower and slower rate and asymptotically approaches (but never actually reaches) its maximum value. It would take an infinite amount of time to actually reach the maximum value since the rate of increase (current) becomes smaller and smaller as time goes by. If we take into account the fact that current and charge both vary with time, the equation obtained by applying Kirchhoff's Voltage Rule around the charging circuit becomes:

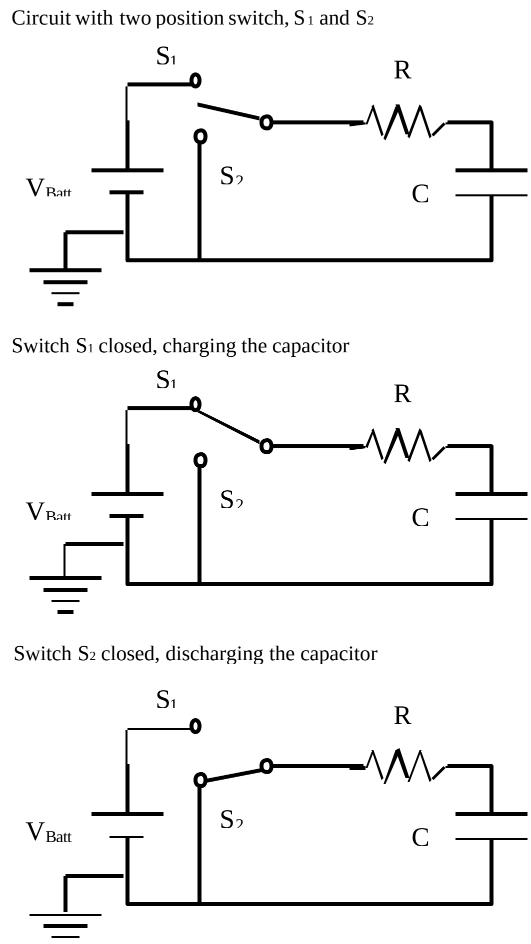


Figure 13: Circuit to charge and discharge a capacitor

<u>Equation 1: Kirchhoff's Rule for a charging capacitor</u>

$$V_{Batt} - V_R - V_C = V_{Batt} - I(t) \cdot R - \frac{Q(t)}{C} = 0$$

The exponentially decreasing current and increasing charge are describe by:

Equation 2(a): I(t) for the charging capacitor

$$I(t) = \frac{V_{Batt}}{R}\left(e^{-t/RC}\right)$$

Equation 2(b): Q(t) for the charging capacitor

$$Q(t) = CV_{Batt}\left(1 - e^{-t/RC}\right)$$

The number *e* (Euler's Number also known as the Natural Logarithm Base) appears in any equation in which the rate of increase or decrease of a quantity depends on the amount of that quantity. In this case the rate at which the capacitor charges depends on the current which decreases as the charge, and hence the voltage on the capacitor, increases. It takes an infinite amount of time for the current to actually decrease to zero or for the capacitor to become fully charged -- this is a property of exponential functions. The exponential behavior is characterized by the *time constant*, t :

Definition of time constant:

$$\tau = RC$$

The time constant has units of seconds -- the larger the product RC the longer it will take the capacitor to charge to any fraction of its maximum value. The time required to charge to increase to $\left(1 - e^{-1}\right)$ of the maximum value is exactly one time constant, RC. Euler's number, e is equal to $2.718281828459045235360287 47 \cdots etc.$

If we wait for until several time constants have passed, the capacitor will becomes nearly fully charged. At that time the current is nearly zero, the voltage on the capacitor is equal to the voltage on the battery, V_{Batt}, and its charge Q_0 is given by $Q_0 = CV_{Batt}$. Now we can change the switch from position S_1 to S_2. The current will flow through the resistor to ground, discharging the capacitor. Around this loop the sum of voltages is now given by

Equation 3: Sum of voltages around the discharging circuit:

$$V_R + V_C = I(t) \cdot R + \frac{Q(t)}{C} = 0$$

The voltage on the capacitor acts to "push" the current but as the current flows the capacitor discharges and the current slows down. Thus the rate of discharge slows as time goes by. Both the current and charge *decrease exponentially* with time:

Equation 4(a): I(t) for discharging capacitor

$$I(t) = \frac{V_C}{R} e^{-t/RC} = \frac{Q_0}{RC} e^{-t/RC}$$

Equation 4(b): Q(t) for discharging capacitor

$$Q(t) = Q_0 e^{-t/RC} = CV_{Batt} e^{-t/RC}$$

The same time constant, $t = RC$ is used to characterize the discharging cycle but now the time RC describes the time it will take the charge to decrease to $1/e = e^{-1}$ of its initial value.

Procedure

Using the breadboard with built in power supplies, **construct the circuit** in Figure 13. You will use capacitance values between 10 and 100μF and resistance values from 50kΩ to 1MΩ. Choose values so the product of R times C is on the order of several seconds. With the switches set in the “open” position, set up the power supply with a voltage of about 5 V. By using the switches at the bottom of the board it is possible to switch between the two configurations. Use a multimeter set as an Ohmmeter to check continuity in your circuit and as a Voltmeter and make sure you understand how the circuit works. Ask for help if you need it.

Preparing the data acquisition hardware: The voltage will be measured using the Universal Laboratory Interface (ULI) with the voltage probes connected to input 1. To drive the ULI, load the program labeled "data logger". Place the voltage leads across the capacitor. By measuring the voltage across the capacitor you can measure charge since Q=CV.

Preparing the plot: It's a good idea to change the plot which comes up on the data logger program. You can change the length of time on the x-axis. Make the time at least ten times longer than your time constant, RC. To do this click twice on the plot just above the x-axis. Change the upper time limit. Set the minimum for the y-axis to be -0.5V and the maximum about 0.5 Volts higher than your power supply. Click on the vertical axis label to select the Port 1 input only. The **Collect** menu can be used to change the ports plotted or the sample rate if needed.

To collect data you should **start with the switch in the grounded position** (S_2 in Figure 13). You can use an extra wire to **ground both sides of the capacitor** to make sure it is fully discharged before you start.

Click **Start** on the program. You should see a horizontal line at about zero volts. After a couple of seconds flip the switch to the charging position. You should see the voltage continually increase. If needed stop and re-adjust both the time scale and data collection rates. The maximum time scale should be about ten times the product of R times C and there should be enough data points per second to give a nice smooth curve. Repeat until you are sure you can make a good data plot.

Once you have adjusted the time scale and data collection rate you can begin taking data.

Start with the switch in the S_2 position and ground the capacitor to discharge it.

Click start on the program and let it run for a while to establish a flat zero volt baseline.

Flip the switch to S_1. Let the program run as the voltage rises.

When the voltage reaches the maximum **let the program run** so that there is a flat line at the maximum voltage --- the charge on the capacitor has now reached its maximum and the current has stopped flowing.

Keep the program running and flip the switch back to S_2. Now the capacitor will discharge. Let the program run long enough to again establish a flat zero volt baseline.

If there is room on your time scale you can continue flipping the switch for additional cycles. Print the graph if desired.

If you have recorded data for several cycles, choose the best looking charge/discharge graph and adjust the range on the time axis so only one cycle is plotted. (For example if the best looking graph is between 75 and 140 seconds then adjust the time axis to plot that range). Print a copy of that graph for each member of your group.

From your graph, **determine the time constant** of the circuit for both the charging and the discharge portions of the curve. From the time constant and the measured value for your resistance, derive the capacitance. (See ***Data Analysis*** section that follows)

IMPORTANT: STOP and analyze your data before proceeding to the next section! You will need to have the data visible on the computer in order to analyze it.

Repeat the data collection for two capacitors in series *and* for two capacitors in parallel. Demonstrate that the sum of the capacitance conforms to the formula derived in class:

Capacitors in Parallel: $C_{equivalent} = \sum C_i = C_1 + C_2 + C_3 + \ldots$

Capacitors in Series: $\frac{1}{C_{equivalent}} = \sum \frac{1}{C_i} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$

Data Analysis

Be sure and complete your analysis as you complete each part of the lab

The "time to charge" or the "time to discharge" is actually infinite since it takes an infinite amount of time to fully charge or discharge the capacitor. The charging and discharging times are therefore *characterized* by a single time t=RC, called the time constant, τ. From the equations for voltage as a function of time we can figure out what the voltage actually is at these times.

When the capacitor is charged its charge increases according to Equation 2 and therefore its voltage as a function of time is given by the equation:

$$\mathrm{V_C(t)} = \frac{\mathrm{Q(t)}}{C} = \mathrm{V_{Batt}}\left(1 - \mathrm{e}^{-t/\mathrm{RC}}\right) \qquad \text{(Charging)}$$

The time constant, τ=RC, is the time when the charge reaches the value given by setting the time t equal to the value RC:

Equation 5: Charge (or voltage) at time t=RC tells you time constant for charging

$$\mathrm{V(at}\ t = \mathrm{RC}) = \mathrm{V_{Batt}}\left(1 - e^{-1}\right) \qquad \text{(Charging)}$$

To find the time constant from the graph simply find the time it takes the voltage (and hence charge) to increase from zero to (1 - e^{-1}) times its final value. You will find this time by analyzing the graph.

When the capacitor is discharging, the charge decreases according to Equation 4 and therefore its voltage obeys the equation:

$$\mathrm{V}(t) = \mathrm{V_{Batt}}\, e^{-\mathrm{t/RC}} \qquad \text{(Discharging)}$$

The time constant is the time when the charge reaches the value given by this equation:

Equation 6: Voltage at time t=RC tells you time constant for discharging

$$\mathrm{V\ (at\ t = RC)} = \mathrm{V_{Batt}}\left(e^{-1}\right) \qquad \text{(Discharging)}$$

To find the time constant from the graph simply find the time it takes the voltage to decrease from its maximum value to (e^{-1}) times that value. You will find this time by analyzing the graph.

To analyze the graph for a charging cycle you need to find the time it takes the voltage on the capacitor to increase from zero to $\left(1 - e^{-1}\right)$ of its maximum value. Have a charging cycle clearly visible on the computer screen. From the **Analyze** menu choose Analyze Data A. When you move the cursor to the plot it becomes a cross-hair and you can read the time and voltages for any data point at the bottom of the plot. Find the time just before the voltage starts to increase. Transcribe these numbers from the computer to the plot you have printed and label this point. Find the maximum voltage on the computer and similarly label this on your graph. Now calculate the voltage $\left(1 - e^{-1}\right) \cdot V_{Max}$. Move your cross-hair until you find the point nearest to this voltage on your graph. Label the voltage and time on your print-out. The time constant for charging is the time between V=0 and V=$\left(1 - e^{-1}\right) \cdot V_{Max}$. Draw vertical lines down from these points on your graph to intersect the time axis. Draw a horizontal double headed arrow $(\leftrightarrow)$on your graph between these two points and label it Δt. Calculate the difference in time and also write this on your graph.

To analyze the time for discharging follow a similar procedure except that now you want the time from the maximum voltage (just as the switch was thrown to start the discharging circuit) until the voltage decreases to

e^{-1} of its maximum value. Find and label the starting point for the discharge cycle when $V = V_{Max}$. Calculate the voltage $e^{-1} \cdot V_{Max}$ and find the corresponding point on the graph. On your graph record the times and voltages found on your computer and find Δt for the discharging cycle. As in the previous case, draw the vertical lines down from these points and label Δt.

Once you have found the time constant, complete the calculations in the data table to compare the measured time constant to the theoretical value. You can use the known value of the resistor to calculate an experimental value for the equivalent capacitance for the capacitors in parallel and series. The theoretical equations for the equivalent capacitance in these two circuits are on page 185.

For your report

For your report, write a cover page with brief introduction and include data table, graphs and analysis. All graphs should have axis and units labeled. The analysis which gives you your time constants clearly shown.

Calculations given in the data table should be shown below the graphs or on a separate page. Summarize your experimental results and discuss errors or discrepancies and their possible sources.

Discussion and Questions

Discuss why the time required to charge and discharge changes with capacitance and resistance (How much charge is required to reach the final voltage? At what rate (current) does the charge move? What controls the current?)

How does the time constant change when the capacitors are in parallel? In series? How does the capacitance change.

Analyze the time dependence of the solutions in Equations 2 and 4.

- Look at Equation 2(b) and Equation 4(b) for the charge (or voltage since VC=Q/C) at time t=0. Substitute t=0 in the equations. Does each equation give the appropriate initial value for a charging or discharging capacitor?
- Look at Equations 2(a) and 4(a) and determine what is happening to the current at time t=0.
- Look at the same equations at very long time (substitute a very large value or imaging what the value reaches in the limit t=infinity). Do the equations reach the appropriate limits at very long times?

Write two paragraphs describing in your own words what is happening to the charge on the capacitor, the voltage on the capacitor, and the current in the circuit as the capacitor is 1) charging and 2) discharging. Comment on whether the time dependence shown by the equations agrees with the expected time dependence of the charge and current in the circuit as it charges or discharges.

Calculus Based Questions: Understanding the equations in more detail.

Equations 1 and 3 describe the charging and discharging of a capacitor. The solutions to these equations are Equations 2 and 4, respectively.

Equation 2(b) describes the charge as a function of time as the capacitor is charged.

- Find the currents for the charging capacitor by calculating the function I(t)=dQ/dt for this case. (Take the derivative of Equation 2 to find I(t).) Compare to Equation 2(a).
- Substitute the function Q(t) of Equation 2(b) and the function for I(t) which you have just found into Equation 1, show that Equation 1 does give zero. This proves that Equations 2(b) is a solution for this circuit.

Equation 4 describes the charge as a function of time as the capacitor is discharged.

- Find the currents for the discharging capacitor by calculating the function I(t)=dQ/dt for this case. (Take the derivative of Equation 4(b) to find I(t).)

By substituting the function $Q(t)$ of Equation 4(b) and the function for $I(t)$ which you have just found into Equation 3 show that Equation 4 is indeed a solution to the voltage equations for a discharging capacitor. (Show Equation 3 does give zero).

Single Capacitor

Applied Voltage V_{Batt} ("Battery"):	Resistance R used	Capacitance C used

From graphs: be sure and label points on graph and show how your found the time constant

Measured Time Constant (from graph);	Charging	Discharging	Average:

Calculations:

Calculated Time Constant (τ =RC):		Percent Diff. (between average and calculated)	

Parallel Capacitors

Applied Voltage V_{Batt} ("Battery"):	Resistance R used	Capacitance s used	C_1	C_2

From graphs: be sure and label points on graph and show how your found the time constant

Measured Time Constant (from graph);	Charging	Discharging	Average:

Calculations:

Calculation for C_{equiv}			
Calculated Time Constant (τ =RC):		Percent Diff. (between average and calculated)	
Experimental C_{equiv} (From τ =RC)		Percent Diff. (between experimental and calc. C_{equiv})	

Series Capacitors

Applied Voltage V_{Batt} ("Battery"):	Resistance R used	Capacitance s used	C_1	C_2

From graphs: be sure and label points on graph and show how your found the time constant

Measured Time Constant (from graph);	Charging	Discharging	Average:

Calculations:

Calculation for C_{equiv}			
Calculated Time Constant (τ =RC):		Percent Diff. (between average and calculated)	
Experimental C_{equiv} (From τ =RC)		Percent Diff. (between experimental and calc. C_{equiv})	

21. *Charging and discharging a capacitor.*

Chapter 5: Magnetism

22. Activity: Magnets and Magnetic Fields

Introduction

In this activity you will investigate the properties of permanent magnets, the magnetic fields they produce and the forces between magnets and other objects.

Equipment

Two Permanent Bar Magnets. *(Make sure they are not so old they are demagnetized*)
Set of 6 to 8 small disc shaped magnets.
Objects of conducting material: Iron, Steel, Copper, Aluminum.
Objects of insulating material: glass or plastic
Compasses String and tape. Iron Filings, paper.

<u>1. Permanent Magnets and Forces</u>

Equipment: bar magnets, conducting and insulating objects.

Permanent magnets interact with each other as well as other objects. Begin with at least two permanent magnets (these should have poles labeled "North" and "South") and other objects from the list above. (Metal objects should include at least iron, copper and aluminum, insulators can be glass and plastic.) Systematically investigate the interaction between the magnets and the other objects.

What happens when you bring "unlike" ends of the magnets together?

What happens when you bring "like" ends of the magnets together?

Compare the magnets to an electric dipole.

Describe the interactions of the magnets with conducting materials. Is the interaction with the different ends of the magnet the same or different.

Describe the interactions of the magnets with insulating materials.

How does the magnetic "behavior" of the materials compare to their electrical properties? Is the comparison simple?

Recall that when a neutral conductor is brought near an electrically charged object (either positive or negative), the electrons in the conductor moved to create an *induced charge* and the neutral conductor is attracted to *either* positively or negatively charged objects. Compare this behavior to the behavior of (some of the) materials in the presence of the permanent magnets.

<u>2. Magnet Orientation</u>

Equipment: two magnets, string, tape, compass

Now you will investigate how a magnet orients itself in space relative to another magnet. Tie a string firmly around the middle of a bar magnet and secure it with tape. Suspend the magnet. Bring a second magnet near and move it around to observe the affect.

Describe the affect.

Does the affect vary with distance?

Next place a compass on the table and observe the affect of bringing a magnet near it. Move the magnet around.

What is the compass needle made of? Which end points where?

3. Magnetic Field Lines

Equipment: Bar magnets , iron filings, paper.

Place a permanent bar magnet on a piece of paper or plastic. Lightly sprinkle iron filings around it. The needle shaped iron filings should orient themselves along the magnetic field lines and so you can create an image of the magnetic field around the magnet.

Sketch the magnetic field lines.

Compare your sketch to the electric field lines around an electric dipole.

Return the filings to their container when you are done.

4. Breaking Apart a Magnet

Equipment: Six to Eight small disc magnets, two bar magnets, pieces of iron or steel.

Investigate the small disc magnets.

a) Does each appear to have both a "North" and a "South" pole? Explain how your observations reveal this.

Assemble the disc magnets into a stack. Investigate its behavior compared to the single bar magnet.

b) How does the stack behave compared to a single bar magnet? Does it have a "North" and "South" pole?

Now divide your magnets into two smaller stacks. Investigate their behavior.

How does each small stack behave compared to either the larger stack or the single magnet?

Is the behavior of a single disc magnet by itself the same as the behavior of a disc when it is in the center of a stack?

Pull the stacks completely apart again.

e) Speculate on what might happen if you continued to divide the magnets in half.

23. Measuring the magnetic field of the earth

Introduction

Magnetic fields, $\boldsymbol{B}$, can be produced by *moving* charges (such as currents) and in turn produce forces on moving charges. In this lab you will measure the magnetic field produced at the center of a coil of wire carrying a current and compare it to the earth's magnetic field.

Equipment

Dip needle, compass, tangent galvanometer (coil), DC power supply, cables, ammeter (or use internal readout on Hewlett-Packard supplies).

Note: steel lab tables and chairs will interfere with the Earth's field. If the lab benches are steel, this experiment must be conducted on the floor away from steel tables and chairs.

Theory

A magnetic moment is the source of the magnetic field. The simplest type of magnetic moment is the *magnetic dipole*: magnetic field lines go from the North pole to the South pole.

The Earth has a permanent magnetic moment. The geographic North Pole of the earth is actually its magnetic "south" pole since the north pole of a compass needle will point North.

Atoms can have small *magnetic moments* which produce small magnetic fields. These are due to both the intrinsic magnetism of the protons, neutrons and electrons and to the circulation of the electrons about the nucleus. In most cases the magnetic moments of the atoms are randomly aligned so that on average there is no net magnetic field for the material.

A permanent magnet is a material in which the atoms are aligned so as to produce a net magnetic field. A simple bar magnet typically has a North and South pole at its ends. Opposite poles attract each other, like poles repel.

A compass is generally a needle shaped permanent magnet mounted on a pivot so that it can rotate to show the direction of an external field. The North pole of the compass is often painted red and will point towards the Earth's North Pole (which is its south magnetic pole) since opposite poles attract.

Magnetic materials are materials that can produce an *induced* magnetic field in the presence of an external field because their atoms are able to realign the internal magnetic moments with the external field. An example is a *ferromagnetic* material such as iron or steel. Ferromagnetic materials are attracted to either pole of a permanent magnet because of their induced magnetic moment. Not all metals are ferromagnetic: Aluminum is not magnetic.

An electromagnet uses a current to create a magnetic field. The direction of the field is determined by the right-hand rule. For a current loop if the fingers of the right hand curl in the direction of the current the thumb will point in the direction of the magnetic field through the inside of the loop.

Procedure

The Earth's magnetic field:

Here you will use a "dip needle" to find the direction of the Earth's magnetic field and calculate its horizontal component. Instructions for the dip needle are found inside its box.

Take the dip needle to a position away from the steel lab tables, which will interfere with this measurement.

First align the protractor horizontally and find North. Note this position for future reference.

Align the dip needle so that the needle and its support are aligned along North. Then rotate the protractor to let the dip needle point downward. The Earth's field actually points downward into the Earth since the position of the magnetic North pole is downward. Record the downward angle that the dip needle makes with the horizontal direction.

Make a sketch of the dip needle and indicate the angle measured. Use the angle to find the *horizontal* component of the Earth's field. Use the value of $5x10^{-5}$ Tesla for the total Earth's field. This is a typical value the local conditions may vary or the building and steel furniture may distort the Earth's field.

The magnetic field produced at the center of a coil

WARNING: TURN DOWN the power supply every time you connect or disconnect leads. DO NOT let the current exceed 5 Amps. (The Hewlett-Packard supplies are limited to 3 Amps)

Measure the radius of the wire in your coil. Set up the coil with the compass at the center of it so that the plane of the coil is aligned North. The compass needle should point North. Rotate the casing of the compass until the North label also points North.

DO NOT TURN ON THE POWER SUPPLY YET. If the power supply does not have an internal current read-out, place the ammeter on its highest setting (for example, 20 Amps). Connect the ammeters so that a lead goes from the + terminal of the power to the Ground or - input of the Ammeter and a second lead goes from the Amp terminal of the ammeter to one of the knobs on the coil --- the ammeter should be *between* the power supply and the coil so that it measures the current going towards the coil. MAKE SURE THAT THE AMMETER IS NOT CONNECTED DIRECTLY ACROSS THE POWER SUPPLY. Connect the other lead from a second knob on the coil (across the terminals marked 15 turns) to the ground on the power supply. If the power supply has a current indicator, you can simply connect it directly to the coil.

Turn up the power supply slowly to test your set-up, do not let the current exceed 5 Amps. As you change the number of turns (coils of wire) used re-test to make sure that you never exceed 5 Amps. (Note: if the current will not increase the supply may be set in a current limiting mode: ask your instructor for help).

Turn the current back down again. Now watch the compass needle as you *slowly* turn up the current (keeping it below 5 Amps). You may need to tap the compass to make sure the needle doesn't stick. Turn it up and down a few times and observe what happens.

TURN THE POWER SUPPLY DOWN and switch the leads. Turn the current slowly up to 3 to 5 Amps while observing the compass needle. What has happened? Make a sketch and record your observations in a section labeled **Observations**.

The direction of the magnetic field produced by the coil is perpendicular to the place of the coil. Thus if the compass needle originally points North and the coil is aligned so that this is in the plane of the coil, you should observe that the compass needle rotates as the current-produced ***B*** (magnetic) field becomes stronger. When the needle has turned 45 degrees the current-produced ***B*** field (B_{coil}) should be equal to the *horizontal* component of the Earth's field (B_{earth}).

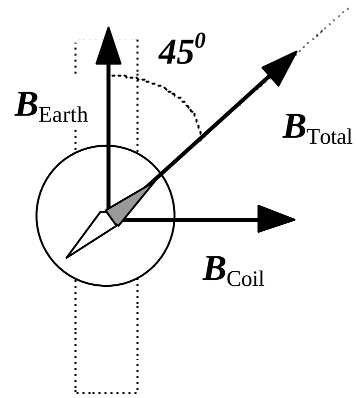


Figure 1: Current in the coil produces a magnetic field which deflects the compass needle.

Carefully adjust the current until the compass needle points 45 degrees from North (this should be either NE or NW). Tap the compass if necessary to make sure it isn't sticking. Turn it up and down a few times to see how reproducible the current is. Make a note of the current used. TURN THE POWER SUPPLY DOWN and switch the leads --- how much current does it take to turn 45 degrees the other way? If the current for each direction is not (nearly) equal then your coil may not be aligned with North.

Once you are sure the plane of the coil is aligned with North, repeat your measurements for the current required to turn the needle 45 degrees 3 times for each direction of current. Turn the current down to zero between each trial and tap the compass to make sure it isn't sticking.

TURN THE POWER SUPPLY DOWN and switch the leads on the coil so you are using 10 turns of wire. CHECK TO SEE HOW FAR YOU CAN TURN UP THE POWER SUPPLY WITHOUT EXCEEDING 5 AMPS. (With fewer turns the resistance is less and so the current will be higher for the same voltage setting.)

Repeat your measurements for the current required to turn the needle 45 degrees each way. Make three measurements for each direction.

Repeat using 5 turns of wire.

Summarize your observations in your report.

Analysis

When the compass needle turns to 45 degrees the field produced by the coil should be equal to the *horizontal* component of the Earth's field.

Preliminary Analysis. (Answer in your report)

How reproducibly could you measure the currents required to turn the needle 45 degrees?

Average the currents measured for the different numbers of turns and for the two directions. Record in the table.

Is the magnetic field required to turn 45 degrees the same in each case? (What is the coil's field when the angle is $\pm 45^0$?)

Calculate 1) the ratio of the current required for 15 turns to the current required for 5 turns and 2) the ratio of current required for 10 turns to current needed for 5 turns. Record in the table.

How does the current required compare to the number of turns?

Theoretically the magnetic field produced at the center of the coil should be

$$B = N\,\frac{m_0 I}{2R}$$

where N is the number of turns of wire in the coil, I is the current and R is the radius of the coil. The constant m_0 is called the permeability of free space and is equal to

$$m_0 = 1.26 \times 10^{-6}\ Tesla \cdot meters\,/\,Amperes$$

The standard units for magnetic field are Tesla. The direction of the magnetic field depends on the direction of the current. The right-hand rule is used: if the fingers curve in the direction that the current moves around the coil, the thumb points in the direction of the magnetic field.

Further Analysis. (Answer in your report)

If ***B*** is constant (and the radius is constant) how must the current, I, change as the number of turns in the coil changes?

Does this agree with your observations? If not exactly, is the trend right?

For each number of turns, use the average current for that number of turns to calculate the theoretical magnetic field produced at the center of the coil.

Compare to the expected value. Is the result within the accuracy with which you could reproducibly measure the currents? If not, discuss possible reasons.

Make two sketches of the coil showing the two directions for the current. Use the right-hand rule to predict the direction of the magnetic field in each case and indicate this on your sketch.

Report

Your report should include a cover page, Introduction, data and observations. Your introduction should include a sketch of your experimental set-up. You should write sections labeled Observations, Preliminary Analysis and Further Analysis answering the questions posed in each section. Answers must always be complete sentences which can be read independently of the handout. For your Conclusion summarize your results and discuss any random or systematic discrepancies in your data.

<u>The Earth's magnetic field:</u>

Sketch: Calculation:

Total B of Earth very approximately = $5x10^{-5}$ Tesla
(local B_{Earth} may vary)
Horizontal component of B_{Earth} = ________________

<u>The magnetic field produced at the center of a coil</u>

Radius of coil = ____________________________

Trial	Number Turns, N	Current - 45 degrees CW	Current - 45 degrees CCW	Average Current, I	Ratio of Measured Current to the Current Measured for 5 turns of wire	Theoretical B_{coil} (Show calculation)
1	5					
2	5				1.00	
3	5					
1	10					
2	10					
3	10					
1	15					
2	15					
3	15					

Calculations:

24. Current Balance: Measuring the Force Between Two Current Carrying Wires

Introduction

Parallel wires transporting electrical current interact with each other. The interaction is strong enough so that the interaction force can easily be determined in an introductory laboratory. The design of this apparatus is based on the experiment that defines the mks unit of electrical current, the ampere (A).

Equipment

- Current Balance
- Fractional Weight Set
- Reversal Switch
- He-Ne Laser and scale
- 3-10A Ammeter
- Vernier Calipers (metric)
- 3-10A Power Supply
- Leads
- Screens to block light and display position

Safety Issues:

Lasers are potentially dangerous. Even a reasonably low power laser can cause damage to your eyesight. Under no circumstances should you direct the laser light toward a classmate or yourself. Always direct the light away from the rest of the class. Reflected light should be intercepted with the screens provided.

Most of the work performed in this lab uses power supplies outputting small voltages (<18V), so shock hazards are minimal. However, it is always important to turn off the current through the circuit when making any modifications to the experiment. This will eliminate any possibility for shock and improve you experimental results since ohmic heating will be minimized.

Before the Lab

Read and *understand* the theory section of this document. Answer the questions at the end of the theory section. Read the procedure section once.

Theory

When a charged particle moves in a magnetic field, it experiences a force known as the Lorentz Force given by the expression

$$\vec{F} = q\vec{v} \times \vec{B} \tag{1}$$

where q is the charge on the particle, v is the velocity of the particle and B is the magnetic field. If the charged particle is an electron and the direction of motion of the charge is perpendicular to the magnetic field (on average), then the cross product reduces to

$$F = evB. \tag{2}$$

Generalizing this expression for a current carrying wire in a magnetic field is accomplished by adding the force on each of the moving charge carriers. In fig. 1, we will count all of the charged particles in the volume of wire $V=AL$, where L is the length of the wire and A is the cross sectional area of the wire. If the density of charge carriers in this volume is n (#/m^3), then the total number of

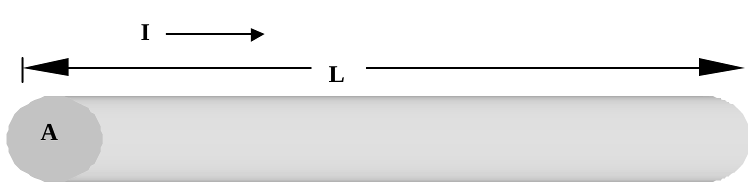


Fig. 1. Schematic diagram of a current carrying wire.

charged particles in this volume is $N=nAL$. Equation 2 can then be modified to give the Lorentz force acting on a length of wire L long

$$F=nALev_dB, \tag{3}$$

assuming that B is uniform throughout the region of the wire. The drift velocity of the charged particles, v_d, has been substituted for the velocity of the electrons in free space.

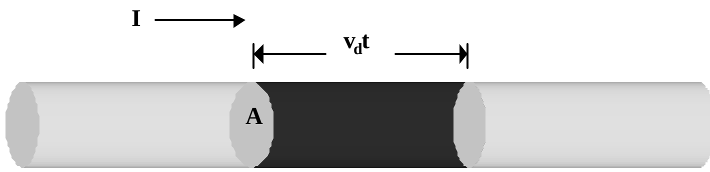


Fig. 2. Schematic diagram of the volume used to calculate the current.

The current is the amount of charge that passes any cross-section of the wire per unit time. In this case, we determine the number of charged particles (number of electrons) contained in the shaded region. The charge in this region passing any cross-section in time t is

$Q=nev_dtA,$

where the parameters are as they were defined above. The current in the steady state is

$$I=dQ/dt=Q/t= I = \frac{dQ}{dt} = \frac{Q}{t} = \frac{nev_d tA}{t} = nev_d A. \tag{4}$$

The force, eq. 3, acting on the current carrying wire can be rewritten by substituting eq. 4 into eq. 3 as

$$F=(nAev_d)LB=-ILB \tag{5}$$

which is the force acting on a wire carrying uniform current I.

If a second wire is close to the first carrying the same magnitude current I, this one will "create" a magnetic field (choosing this wire to create the magnetic field is arbitrary, we could have chosen the other wire as the magnetic field generator). If the wires are exactly parallel, the magnetic field "sensed" by the first wire is uniform and B is a constant along the length of the wire. The magnetic field created by the second wire at a distance r from the center of the second wire is by Ampere's Law

$$\oint \vec{B} \bullet \vec{dl} = m_o I \ .$$

Constructing a Gaussian surface as shown in fig. 3, the path integral reduces to $2pr$ and I is the current in the wire. The uniform magnetic field is given by

$$B(2pr) = m_0 I \Rightarrow B = \frac{m_0 I}{2pr}. \tag{6}$$

Placing this result in eq. 5 yields

$$F = ILB = IL\frac{m_o I}{2pr} = \frac{m_o L}{2pr} I^2 \tag{7}$$

where L is the length of the wire and r is the separation between wires (center to center distance). This is the expression we will verify in today's laboratory.

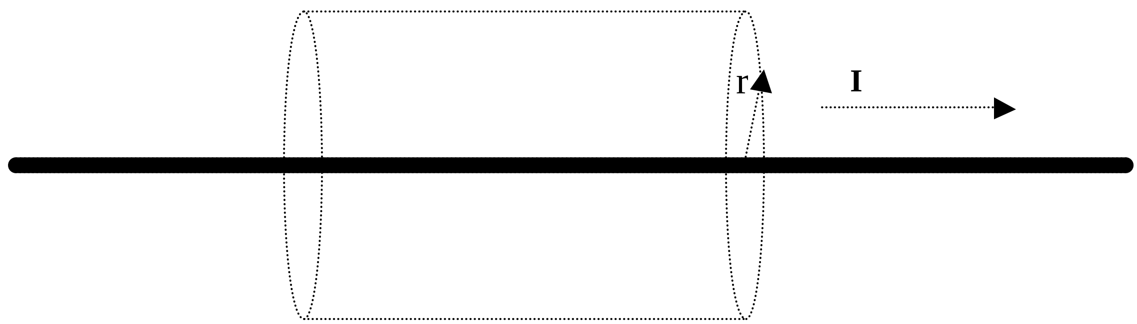


Fig. 3 Image showing the "Gaussian" surface used to determine the magnetic field from Ampere's Law.

Prelab Questions:

If the force between two current carrying wires has increased by a factor of four and the wire has not changed its position, by how much has the current in the wires increased?

If the force between two current carrying wires has decreased by a factor of four and the current has not changed, by how much has the distance between the wires changed?

The magnetic field generated by the current in the stationary wire is a constant. The direction of the current is from left to right in fig. 4. As shown in fig.4, with the movable wire running parallel to the stationary wire, which direction is the magnetic field at the position of the moveable wire?

Procedure:

Setup the simple circuit as shown in Fig. 4. The switch will allow reversal of the current in the moveable wire resulting in a change in the sign of the force. The battery should be replaced with the HP dc power supply provided.

It is very important that lead wires connected to binding posts on the balance leave them at right angles with the conductors that are part of the frame. The circuit shown in fig.4 provides the configuration for a repulsive force between the two parallel bars as required in the experiment. By suitably interchanging the wires connected to the balance, the bars may be made to attract each other.

Caution:

The Earth's magnetic field can lead to errors. This problem can be avoided by orienting the conducting bars parallel to the earth's field.

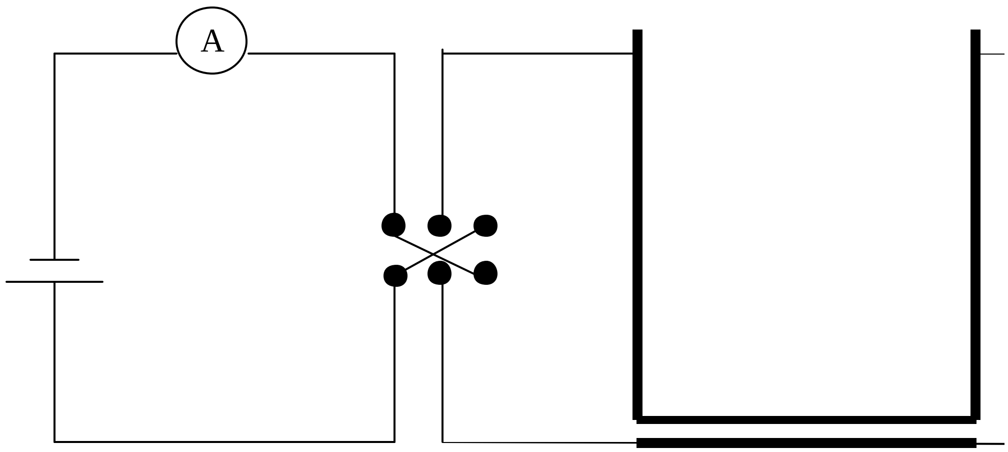


Fig. 4 Schematic of circuit to be constructed with current reversal switch

Remove the frame from the balance and clean the knife-edges, bearings, and the two parallel bars carefully to prevent sticking. The knife-edges have been honed and checked for burrs at the factory. Further honing by the user should be avoided. Be careful to protect these pivot surfaces from accidental scratching. Use only the lifting arms to place and remove the balance assembly from the bearing posts.

Place the balance directly on the laboratory bench. Adjust the leveling screws to position the base firmly. Replace the frame and adjust the counter weight behind the mirror until the frame oscillates freely and comes to rest with the front horizontal bar a few millimeters above the stationary bar. Adjust the counter poise below the mirror until the period of oscillation of the frame is 1 to 2 seconds. It should come to rest in 10 to 15 seconds when the poles of the damping magnets are about 2 mm apart.

To check the two conducting bars for alignment, place a coin on the scale pan to bring the bars into contact without distortion. Thumbscrews on each front post permit either end of the lower bar to be raised or lowered. Similar thumbscrews on each block at the rear permit either end of the upper bar to be moved forward or backward. The two bars should be aligned as accurately as can be determined by the unaided eye when viewed from the front and from the top. When viewed from the front, with a white paper behind the bars, the two bars may appear to be slightly bent. If this is serious, correct by gently bending one bar or the other by hand until both appear to be straight. It is difficult to get them so straight that no light may be seen between them, and this is not essential for good quantitative results. Nevertheless, the bars are rather easily bent and this inspection should be made before every trial. In general, the bars should always be handled gently and as little as necessary.

Setup the laser and scale 1to 1.5 meters from the mirror. Remove the coin from the weight pan and record the rest point indicated by the position of the laser spot on the scale. Note that a screw behind the mirror permits its angle of tilt to be adjusted so that spot height reading is convenient. Engage the beam lift gently; the release it and again record the rest point. If it deviates from the first observation, the knife edges may not be clean, the base or table may be unsteady, or the balance or laser may have been jarred.

Determine the length of the wire to be used in the calculation of the force. Record this value as L_o.

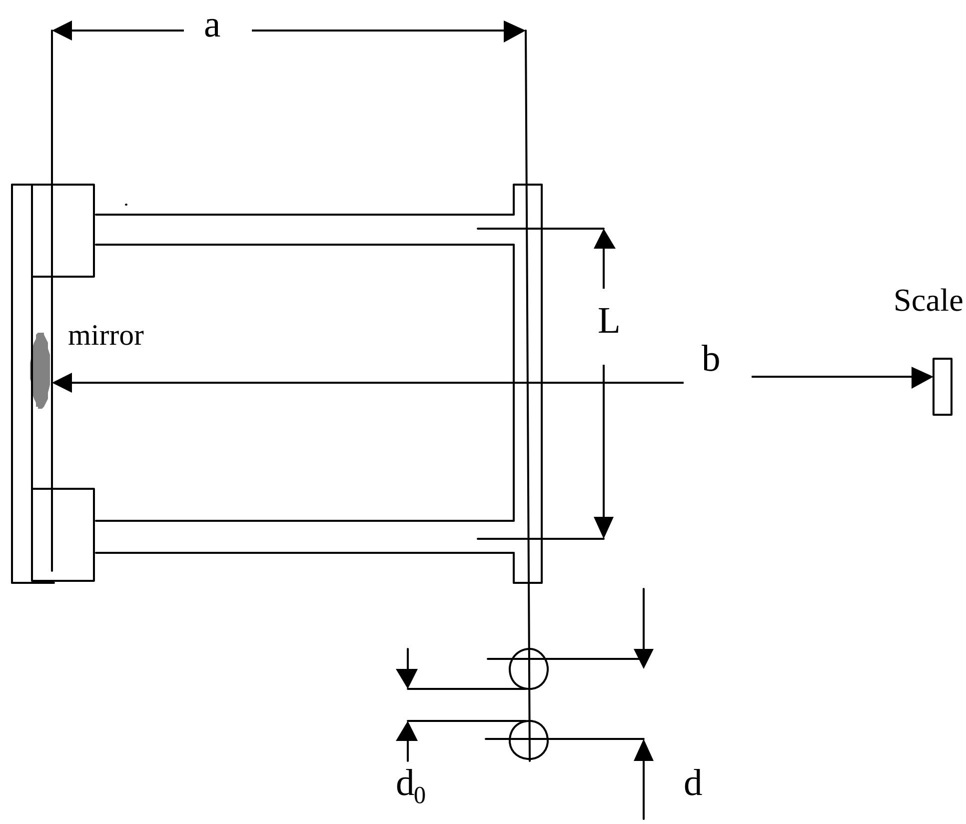
Measure the distance from the pivot to the center of the front bar on each side and record the average of these two as a_o.

Measure the distance from the mirror to the scale and record this number as b.

The separation of the two bars at equilibrium is determined in the following the manner. The scale reading at equilibrium is noted. Then the upper bar is depressed (by placing a coin on the scale pan) until it is in contact with the lower bar, and a new scale reading is noted. Geometry will show that the separation is given by (ask your instructor if you should derive this expression):

$$d_o = a_o D/2b$$

where D is the difference in position on the scale.



The center to center distance d is the sum of d_o and the two radii of the current carrying rods, i.e. $d=d_o+r_1+r_2$. The radii should be measured with a vernier caliper.

Caution:
The purpose of the beam lift is to relocate the knife-edges to their proper positions on the bearing posts so that the front moveable conductor will always be in the same vertical plane as the fixed conductor below it. Lifting the beam each time a weight is added or removed avoids the risk of jarring the beam and damaging the knife edge.

Begin a trial by adding a weight to the pan. Increase the current until the beam returns to its equilibrium position, and read the ammeter. Change the weight on the pan and again adjust the current and read the ammeter. Continue this process until you have recorded at least 10 values between 10 and 150 mg. (Do not exceed 10A. If the current must be increased to more than 10A, decrease the weight. Higher currents will tend to drift due to ohmic heating of the wire increasing the resistance of the rod.)

Plot the weight on the plate (mg) versus the current squared. Perform a linear fit and determine the slope of the line. This slope is related to measured quantities and $\mu_o = 4\pi \text{ x } 10^{-7}\text{H/m} = 4\pi \text{ x}10^{-7}\text{Vs/A}$. Compare your slope to the predicted value.

For a second trial, change the couterpoise slightly to make the separation of the conducting bars slightly different. Record the scale readings at contact and at equilibrium, and take another series of readings.

Data Tables

	Trial 1	Trial 2
Mirror to scale, b		
Radii of bars, r_1+r_2		
Lever arm, a_o		
Length of upper bar, L		
Scale reading at contact		
Scale reading at equilib.		
Difference, D		

Trial 1:

Force , mg (N)	I(A)	$I^2(A^2)$	F/I^2

Trial 2:

Force , mg (N)	I(A)	$I^2(A^2)$	F/I^2

Summarize the experiment and provide samples of all calculations.

25. Investigation of Magnetic Fields due to Current Loops

Any electrical current has a magnetic field associated with it, no matter how small the current. Electrical currents transporting information to and from the human brain are detected by the magnetic field they generate. In the first part of this lab, we will use the computer to see how a current loop can generate a magnetic field and the direction of that magnetic field.

Equipment

Computer programs "Ampere" and "Coulomb" ULI interface magnetic field probe	Bar magnets Compass meter stick and support (rod, clamp)	Coil with 5, 10, 15 turns of wire Small coil with two separate coils Graph paper	Large green DC supply, Banana plug leads Ammeter (5 amps)

Part I: Computer Simulation.

Single Current Loops

Open the application "Ampere". A blank window will come up and a window containing a choice of current levels. Choose "plus 1 current". Under the menu "**Field Lines**", choose "**fine**". Click at one position in the empty window. You will see an "x" and a number both surrounded by a circles. This represents the cross section of a current loop. If a circular current loop were intersecting the computer screen it would pass through it at these two points. The "x" represents current going into the screen, number the "Amps" of current coming out. Under the "**control**" menu, choose "**start**".

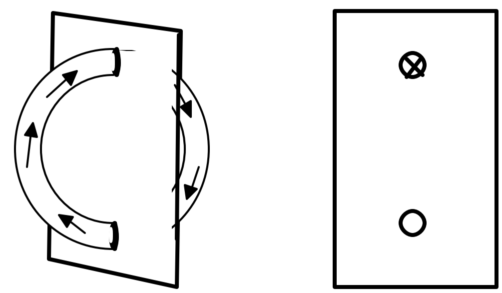


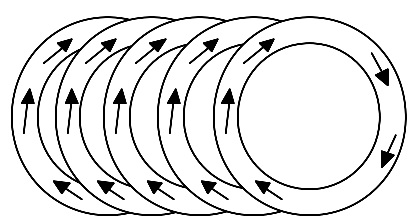
Figure 2: Simulation of current loop intersecting computer screen

Magnetic Field from a +1 current loop
Describe and draw what you see.

Choose "plus 3 current" and replace the "plus 1 current". Make the size of the loops similar. (Use the Edit menu to remove current loops). Run the program.

Magnetic Field from a +3 current loop
Describe the qualitative differences between the plus 1 and plus 3 currents:

Magnetic field from a solenoid



Create a horizontal series of five identical parallel current loops about 1 in diameter spaced ½ inch or less apart. (If you click on the screen ½ inch above the center a dot is drawn both above and below to represent the loop.)Again, choose "**start**" under the "**control**" menu. (This will take longer to run)

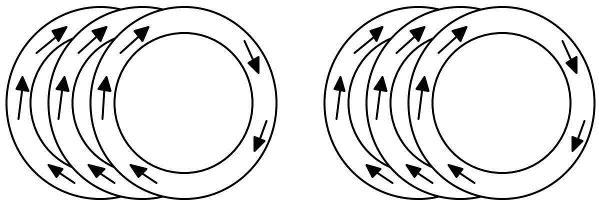
Magnetic field from a solenoid
Sketch what you see here. Indicate a region where the lines of field are nearly parallel.

Directions of magnetic fields due to currents

To determine the direction of the magnetic field, use the "right-hand-rule". Taking your right hand, cup your fingers in the direction of the current flow and extend your thumb. The thumb indicates the direction of the magnetic field inside the current loops.

Add arrows indicating the direction of the magnetic fields to your sketches.
Continue to include arrows for your subsequent sketches.

Magnetic fields for two groups of rings



Generate two adjacent groups of loops (each group containing a least two current rings) which are separated by a distance significantly larger than the spatial extent of a single group.

Magnetic Fields for two groups of rings
Run and sketch the result.

Repeat after reversing the current in one of the groups of currents rings.

Magnetic Fields for two groups of rings (opposite currents)
Sketch the results.

Part II. Magnetic Fields around a bar magnet .

Equipment needed: two bar magnets, compass.

Review of electric dipoles

Open the program called “Coulomb” and use it to draw the field lines around an electric dipole (a charge +3 and a charge -3 spaced a distance apart. (This program is similar to “Ampere”. Use the “Quick Sketch” mode to draw the lines more quickly). Make the spacing similar to the spacing of your current loop.

Electric Field from an electric dipole
Once you have succeeded, sketch what you see.

Investigation of Bar magnets: how do bar magnets compare to electric dipoles?

You should have two bar magnets, each with an end marked North and South. Have all members of your group study the behavior of the two magnets. Use a compass to investigate the shape of the field around the magnets (the compass needle will point in the direction of the field.

Discussion
How is a bar magnet like an electric dipole? How does it differ? Use the compass to see if the field lines are similar.

Part III. Magnetic field due to coils of wire (experiment)

You will use the magnetic field probe and the ULI interface to measure the "z" dependence of the magnetic field along the axis of a coils. The magnetic field probe uses the "Hall Effect", which is described in your book.

You will use the equation for the magnetic field produced along the axis of a coil:

$$B(z) = N\frac{m_0}{2}\frac{IR^2}{\left(R^2 + z^2\right)^{3/2}} \qquad \text{(Equation 1)}$$

where N is the number of turns in the coil, I is the current, R is the radius and z is the distance along the axis of the coil, and $m_0 = 4p \times 10^{-7}\ T \cdot m/A$. The derivation of this formula is sketched out in the Appendix.

Set up the large diameter coil with the power supply and ammeter so that about 5 Amps of current flows through the maximum number of turns. Set up the meter stick so that you can move the probe along the line through the axis of the coil. You will measure the magnetic field as a function of z along this axis, taking data every 2 cm.

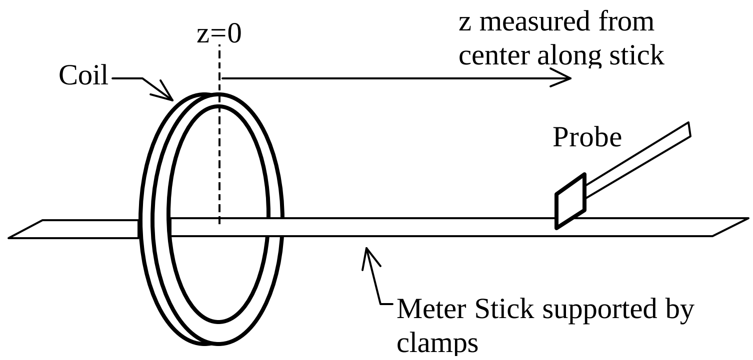


Figure 3: B(z) is measured along the axis of the coil

Safety Note: If your coil or any part of your apparatus becomes hot or has a burning smell, turn of the current immediately and contact your instructor.

<u>Procedure for measuring the magnetic field, B.</u>

Set up the ULI interface and the magnetic probe. Under Physics Applications in the ULI folder and then in the Data Logger Folder find a folder called "Experiments" Open the Magnetic Probe folder and the file for reading fields in milliTesla with low sensitivity. The switch on the field probe should also be set to low sensitivity.

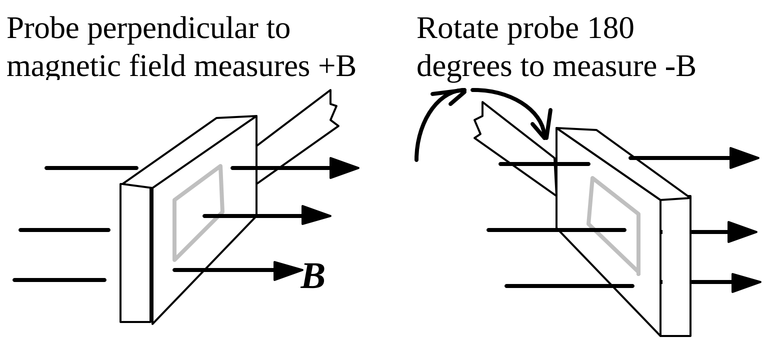


Figure 4: Magnetic Field probe must be rotated in field to measure +B and -B

The magnetic field probe is only sensitive to <u>perpendicular</u> magnetic fields. It will read the component of the field along the direction perpendicular to the probe. Thus when the probe is rotated 180^0 it registers a reading from +B and -B.

Change the scale on the time axis of the program so that data can be collected for five minutes (300 seconds). **Start** the program and rotate the probe in the magnetic field to observe the variation in the measured field as the direction of the probe changes. The maximum and minimum readings are +B and -B. If there were no offset of the zero reading they would be equal except for opposite signs. Change the scale on the axis so that the readings of the probe fit within the range and are readable.

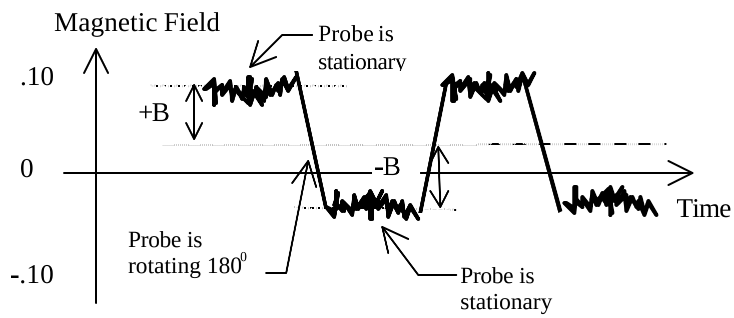


Figure 5: Probe must be rotated 180⁰ to compensate for any offset of the zero reading.

For a reading on the magnetic field first hold the probe aligned with the field for a few seconds to get a reading for +B. Next rotate the probe 180 degrees and then hold it for a number of seconds to get a reading for -B. The results should look similar to Figure 5.

Data can be read from the graph by selecting **Analyze Data A** and then clicking and dragging to select a range of data. The statistics command will then tell you the average (mean) value for that range. Obtain readings for +B and for -B for each data point.

Find the actual value of B for each data point. For example, if due to the offset from zero, +B reads 0.120 mT and -B reads -0.046mT, then the true value of B is $(0.120-(-0.046))/2 = 0.166/2 = 0.082$ mT . The offset from true zero in this case is +0.037 since that is the amount the experimental values of +B and -B are both offset from the true value.

Data collection and analysis

Make a table and record the values for z, +B, -B and the true value for B(z) determined from +B and -B for each value of z (distance from the center).

Be sure to obtain a good value for B(0) at z=0 in the center of the coil.

For each data point calculate the ratio B(z) to B(0), where B(0) is the value at the center of the ring.

Magnetic Field as a function of z

Make a sketch of your experimental setup, showing how z is defined and record the dimensions of the coil. Your sketch should show how "z" is defined. Submit your data table and a plot of B(z)/B(0) versus z. From the formula given (Equation 1) show that $\dfrac{B(z)}{B(0)} = \dfrac{1}{\left(1+\left(\dfrac{z}{R}\right)^2\right)^{3/2}}$. Plot this function and compare to your experimental results.

Part IV: Small coils

First set up a simple circuit involving the smaller coils of wire. Your circuit should have a power supply and ammeter which allows current to flow through the coil. The probe can be inserted in the access port between the coils.

<u>Small Coil</u>

Measure the field at the center of the small coil for a current of 5 Amps. Record both the numbers measured for +B and -B and the calculated B=(+B - -B)/2. Sketch the circuit and the coils, record their dimensions, separation and radii. Use Equation 1 and the dimensions you've measured to make a calculation for the field at the center of the coil. Compare to your measurement.

<u>Field as a function of current</u>

Measure the magnetic field at currents of 1 A to 5 A in increments of 1 A. Plot B field versus current and determine the slope of the plot. Compare to the expected value for the slope.

Appendix: Magnetic Field of a Coil

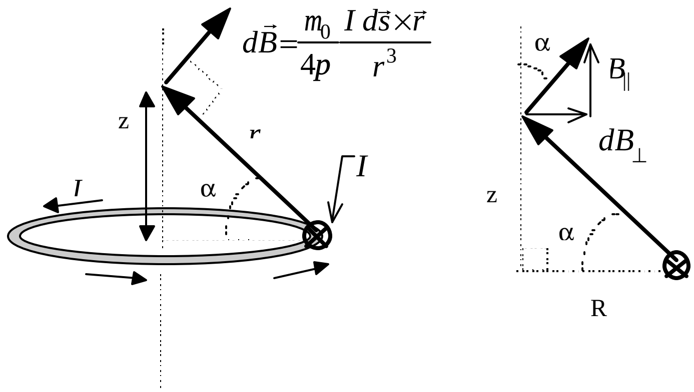


Figure 6: Magnetic field due to a small piece of a circular current loop.

The magnetic field produced at a given point in space by a small piece of current of length ds is given by

$$d\vec{B}=\frac{\mu_0}{4\pi}\frac{Id\vec{s}\times\vec{r}}{r^3}$$

where I is the current, the vector $d\vec{s}$ points in the direction of the current, $\vec{r}$ is the vector pointing from the piece of current to the point in space, and μ_0 is a constant given by

$$\mu_0 = 4\pi\times10^{-7}\,\tfrac{T\,m}{A}$$

Note that because of the cross-product the direction of the vector $d\vec{B}$ is perpendicular to both the direction of the current and the vector $\vec{r}$. In this section you will derive the magnetic field produced by a circular loop of current at a point along the axis running through the center of the loop.

Figure 6 shows the magnetic field due to one piece of the circular current loop at a distance z along the axis through the center of the loop. We consider the piece of current going directly into the paper.

If the radius of the loop is R, find the distance r in terms of R and z.

Find the cosine and sine of the angle, α , in terms of R and z.

Consider the cross product $d\vec{s}\times\vec{r}$. For the piece indicated the current points into the paper and so does the direction of $d\vec{s}$. Since $d\vec{s}$ and $\vec{r}$ are perpendicular to each other, show that the magnitude of the magnetic field can be written as $|dB|=\frac{\mu_0}{4\pi}\frac{Ids}{r^2}$

Now let $d\vec{B}$ be broken into two components, $dB_{||}$ and $dB_{\perp}$, as shown. Show that because $d\vec{B}$ is perpendicular to $\vec{r}$ these components are given by $dB_{\parallel}=|dB|\cos\alpha$ and $dB_{\perp}=|dB|\sin\alpha$. (*Hint: consider which angles add to 90 degrees or to 180 degrees.)*

Now consider what happens when we add up the magnetic field due to all the pieces of current around the whole loop. Imagine taking the piece shown in the figure and letting it rotate around 360 degrees to account for all the current in the loop. Which component of the magnetic field ($dB_{||}$ or $dB_{\perp}$) will add? Which component will cancel?

Since one component cancels we only have to integrate one of the two components to find the total magnetic field due to the loop. Use your results ($\cos\alpha$ or $\sin\alpha$ and r in terms of z and R) to show that the magnetic field can be written as

$$B=\int dB_{\parallel\text{ or }\perp}=\frac{\mu_0}{4\pi}\frac{IR}{\left(R^2+z^2\right)^{3/2}}\int ds=\frac{\mu_0}{4\pi}\frac{IR}{\left(R^2+z^2\right)^{3/2}}\cdot 2\pi R=\frac{\mu_0}{2}\frac{IR^2}{\left(R^2+z^2\right)^{3/2}}$$

where the integral over all the pieces d*s* gives the total circumference of the loop.

Radius of Large Coil:________________________

Data Table

Data				Experimental	Theoretical
z (cm)	+B	-B	B(z)=(+B- -B)/2	B(z)/B(0)	B(z)/B(0)
0				1.00	1.00

26. Magnetic Moment of a Permanent Magnet

Introduction

Although the permanent magnet is a familiar object to most beginning physics students, their understanding is often clouded by misconceptions. In these experiments the student will learn how a magnet experiences a torque in a uniform magnetic field and a force in a magnetic field gradient. Using these concepts they will be able to determine the magnetic moment of magnets using two different techniques.

Equipment

Magnetic Torque MT 1-A apparatus	Ruler	Balance	Calipers
Test Tube holder	Rod stand, clamp		

Before the Lab

Read the sections in your text describing torque. You should be familiar with the formula for the magnetic field produced by a current in a ring along its axis of symmetry.

Theory

<u>Torque on a magnetic dipole.</u>

When a force is applied of a rigid object it can result in a rotation if the torque $\vec{t} = \vec{r} \times \vec{F}$ is non-zero. An object with a magnetic dipole moment, $\vec{m}$, in the presence of a magnetic field, $\vec{B}$, experiences a torque given by the vector product:

$$\vec{t} = \vec{m} \times \vec{B}\,.$$

If the magnetic field is uniform, there will be no net force on the magnetic dipole. This means that there will be no net acceleration but that the object may rotate about its center of mass or pivot. One example of a magnetic dipole rotating in a magnetic field is the familiar compass needle, which rotates to line up with an external magnetic field.

<u>Force on a magnetic dipole in a non-uniform field.</u>

If the field is *not* uniform, however, there will be a net force. This is easily understood if you imagine a magnet made of two magnetic poles of the same magnitude, north and south. (The two poles form a dipole!). Imagine placing this magnet in a uniform external magnetic field. The north pole will experience an upward force and the south pole will experience an equal downward force: the net (total) force is zero. The magnet may rotate (like a compass needle) but it will not accelerate. Now consider a magnetic field pointing up as before but increasing in strength as you move up. In this magnetic field the north pole will experience an upward force that is greater than the downward force on the south pole. The magnet as a whole will tend to accelerate upward. It can be shown that the magnitude of the net force on a dipole in a non-uniform magnetic field is:

$$F_z = m\frac{dB_z}{dz}$$

The derivative, $\frac{dB_z}{dz}$ is called the *gradient* of the field. The more quickly the field changes as a function of distance, z, the larger the gradient will be. The gradient is equal to $\frac{\Delta B_z}{\Delta z}$ for a field changing at a constant rate.

Magnetic torque and gravitational torque

In the Magnetic Torque apparatus, a white "cue ball" contains a cylindrical permanent magnet at the center. The ball has a dipole moment vector $\vec{\mu}$ that points in the direction of the ball's handle. The ball is placed in an external uniform magnetic field, $\vec{B}$, which points upward. Thus the ball's magnetic moment makes an angle θ with the field as shown in Figure 7. The ball will experience a torque $\vec{\tau} = \vec{\mu} \times \vec{B}$ which is equal to $\mu B \sin\theta$. This torque will tend to rotate the handle up. This torque can be canceled if it is balanced by the torque produced by a small weight $w = mg$ acting on the axis of the handle plus a constant torque ("C") due to the weight of the aluminum rod. The distance from the center of the ball to where the weight is hung is the lever arm, $\vec{r}$. The weight therefore produces a torque given by $\vec{\tau} = \vec{r} \times \vec{w}$ which is equal to $rmg\sin(180 - \theta) = rmg\sin\theta$. The right hand rule tells us that this torque points opposite to the one produced by the magnetic field on the dipole. Thus, for equilibrium (no rotation) it is necessary that

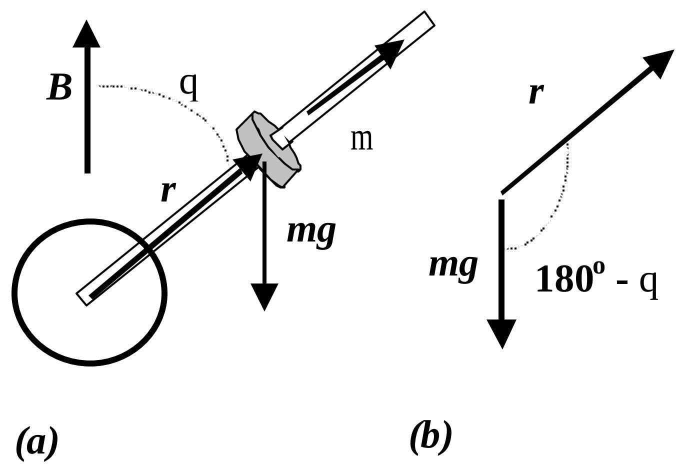


Figure 7: Directions of magnetic field, *B*, magnetic moment, μ, weight, *mg*, and lever arm, *r*.

$$\mu B = rmg + "C". \qquad \text{(Equation One)}$$

Uniform magnetic field (used in torque method)

The magnetic fields of interest to us are those at the center of the apparatus where the magnets will be placed.. The magnetic field of the Earth can be ignored in this experiment, as it is significantly smaller than the typical field applied in this experiment. The source of the externally applied field are the two current carrying coils wire of effective radius $R = 0.109$ m separated by an effective distance $d = 0.138$ m. When the two coils carry current in the *same* direction, the magnetic field at the center is given by:

$$B_z = \mu_o I R^2 \frac{1}{\left(R^2 + d^2/4\right)^{3/2}} \qquad \text{(Equation Two)}$$

Here the z axis is the "upward" direction so we call the magnetic field B_z.

Note : Do NOT confuse the fundamental constant $m_o = 4p \times 10^{-7}\ \mathrm{T \cdot m/A}$, with the dipole moment of the ball, m , which we are trying to measure.

Non-uniform magnetic field (used in force method)

When the two coils carry currents in the *opposite* direction, a magnetic field gradient at the center is set up. It can be shown that the magnetic field inside the coils is given by:

$$B_z = \frac{m_o I R^2}{2} \frac{1}{\left(R^2 + z^2\right)^{3/2}} - \frac{m_o I R^2}{2} \frac{1}{\left(R^2 + (d-z)^2\right)^{3/2}}$$

where z is the distance from the center of one of the coils as measured along the z axis ("up"). Note that the center is therefore located at $z = d/2$ where $B_z = 0$. Taking the derivative of the above expression respect to z and evaluating it at $z = d/2$ we have an expression for the gradient:

$$\frac{dB_z}{dz} = \frac{3 m_o I R^2 d}{2} \frac{1}{\left(R^2 + d^2/4\right)^{5/2}} \qquad \text{(Equation Three)}$$

This expression is exactly true only at the center, $z = d/2$, but for the displacements from center used in this experiment the gradient is nearly constant and this equation will be used for all values of "z".

Magnetic force and spring force (used in the force method)

The plastic tower apparatus contains a magnet hanging on a spring. If the spring is stretched by a distance "Δx" then the force applied by the spring is given by Hooke's Law:

$$F = -k \cdot \Delta x$$

where k is the elastic constant (also called the spring constant) for the spring. The elastic constant can be determined by hanging weights from the spring and observing how much each weight stretches the spring. When the force of the spring balances the weight a graph of weight, mg, versus displacement distance, Δx, should fit a straight line according to the equation.

$$mg = k \cdot \Delta x \qquad \text{(Equation Four)}$$

If the magnet is placed in the field gradient described above then suspended magnet experiences a net force, $F_z = m \frac{dB_z}{dz}$, as previously discussed. This force will stretch or compress the spring. When the force exerted by the spring on the magnet is balanced by the force on the magnet due to the field gradient, we have the equilibrium condition:

$$k \cdot \Delta x = m \frac{dB_z}{dz} \qquad \text{(Equation Five)}$$

where k is the spring constant and Δx is the displacement of the spring from equilibrium. Thus once k is known, we can make a graph of Δx versus the field gradient, $\frac{dB_z}{dz}$ to determine the magnetic dipole moment to the known elastic constant.

Procedure

You will measure the magnetic moment of two magnets. The magnetic moment of the magnet inside the cue ball will be measure by the torque method using the theoretically relation $mB = rmg$. The magnetic moment of the magnet hanging on the spring will be measured by the force method using the relation $k \cdot \Delta x = m \frac{dB_z}{dz}$.

Torque method

First measure all the constants that are involved in the experiment. Use a balance to determine the mass of the weight, the caliper to measure the diameter of the ball (convert to radius, R_B), and the ruler to measure the length of the ball's handle (L).

Insert the aluminum rod in the ball's handle. Make sure that the side of the rod that has a small magnet is the one that is inserted. Also there should be a small plastic ring that can be moved along the rod and plays the role of a weight.

Turn on the power supply and the air. Keep the field gradient and the strobe light off. Set the direction of the magnetic field up. For small currents the gravitational torque is greater than the magnetic torque, so start with a current of about 2.5 A. Record this in the first column of your data table. With that current, adjust the position of the weight until the rod remains stationary at about 90 degrees with respect to the vertical. Take the ball off the air bearing, turn the current down to zero and measure the length from the end of the handle to the position of the center of mass of the weight. Record this number, ℓ, in the data table. Add to this value the length from the center of the ball to the end of the handle. The resulting value is the lever arm, $\boldsymbol{r}=R_B+L+\ell$, of the weight. Record $\boldsymbol{r}$ in your data table for this current.

Repeat the above steps for at least six different currents up to 4 A.

Analysis

For each trial, calculate the magnetic field according to Equation Two and the value of the lever arm, $\boldsymbol{r}$. Show sample calculations. (Hint: factor out all the constants from equation two and just calculate them once!)

Using a computer graphing program (see Appendix), make graphs of $\boldsymbol{r}$ versus $\boldsymbol{B}$. Remember that when you are asked to plot "y versus x" then the "y" should be one the vertical axis. Find the slope and intercept of your graph. From the slope of this graph you should be able to calculate m.

Questions

Why is the y-intercept not zero? (Hint: the aluminum rod also provides torque. What is the effect of its mass and length.)

Supposed you choose and angle between the rod and the vertical to less than 90 degrees, what changes in your analysis will be required?

Force method

1. Place the plastic tower on top of the air bearing with the air turned off. Take the cap of the tower, and insert the rod with the spring and suspended magnet into the hole in the cap. Place the cap on the top of the tower. Adjust the length of the rod so that the suspended magnet is in the center of the coils, that is, where the mark on the magnet is line up with the 0 cm mark on the wall of the tower.

Set the field gradient off and turn up the current. Now keep and eye on the magnet and change the field direction. Write down observations.

Turn the current down to 0.5 A and turn the field gradient on. If the magnet moves down change the field direction. Measure the position x of the center of the magnet. Now increase the current by increments of 0.5 A and measure the respective values of Δx. The maximum current should be 4 A.

Next, the spring constant, k, should be determined. Take the spring with the magnet out of the tower. Use the support stand to hold the spring-magnet system. Attach to the magnet one small ball of about 1 gram (=.001 kg) and record the position of the magnet. Repeat this until you have used all the balls.

Analysis

Plot the data of the spring displacement Δx versus weight mg of the balls hanging on it. From this data calculate the spring constant k. The units of k should be Newtons/meter.

Using Equation Three calculate the field gradient, $\frac{dB_z}{dz}$, for the different currents used.

Plot the displacement Δx versus the field gradient. From the slope of this data calculate the magnetic moment of the hanging magnet.

Questions

Should the y-intercept for the plot of the displacement Δx versus the field gradient be zero? Why may or may not it be zero in your plot?

What, in your opinion, is the major source of uncertainty when you try to measure the magnetic moment of the magnet inside the sphere using this method?

Compare your result to the result obtained in the previous method.

Torque method

Coil Dimensions: d = 0.138 m R = 0.109 m

(Diameter of ball):			
Radius of ball, R_B:		Mass of weight, ***m***	
Length of handle, *L:*		Weight of weight, ***mg***	

Data (make sure all units are MKS: meters, kilogram, Newtons, Tesla, Amperes, etc.)

Current	I							
Distance along rod	ℓ							
Lever Arm	r							
Calculated Magnetic field (Equation Two)	B							

Results of Analysis

$slope = \dfrac{m}{mg}$		units
m		units

Note: $m_o = 4p \times 10^{-7}\ \mathrm{T \cdot m/A}$, $g = 9.8\ \mathrm{m/sec}^2$

Show sample calculations below. Attach clearly labeled graph:

Force method

Coil Dimensions R = 0.109 m d = 0.138 m

Data (use MKS units)

Current	I								
Displacement	Δx								
Calculated Field gradient (Equation Three)	$\frac{dB}{dz}$								

Mass of little balls = 1 gram = .001 kg

Data to determine Spring constant (use MKS units)

Weight	mg								
Displacement	Δx								

Results of Analysis

k		units
$slope = \frac{m}{k}$		units
m		units

Show sample calculations and attach clearly labeled graphs.

27. Worksheet: Electromagnetic Induction

Induced *EMF*: $\varepsilon = -\dfrac{d\Phi_B}{dt}$ Current $I = \dfrac{\varepsilon}{R}$

Magnetic Flux: $\Phi_B = \int \vec{B} \bullet d\vec{A} \quad = \vec{B} \bullet \vec{A}$ for uniform field

$\vec{B} \bullet \vec{A} = |B||A| \cos\theta_{BA}$

$$-\frac{d\Phi_B}{dt} = -\left(|A| \cos\theta_{BA} \frac{dB}{dt} + |B| \cos\theta_{BA} \frac{dA}{dt} - |B||A| \sin\theta_{BA} \frac{d\theta_{BA}}{dt} \right)$$

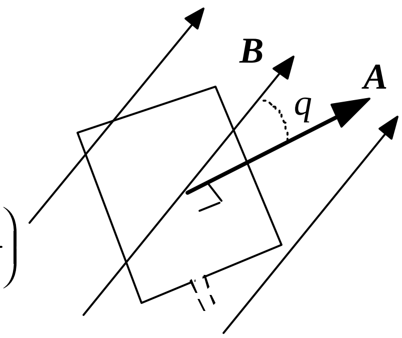


Figure 8: Changing Magnetic Flux creates an induced *EMF*

Changing magnetic flux through any closed loop creates an induced *emf* in that loop. The magnetic flux can change either because the magnetic field changes, the area of the loop changes, or because the angle between the area and the field changes. The following cases illustrate some examples.

<u>Problem 1:</u> A magnetic field of 0.30 T goes through a circular loop of radius R=10 cm so that it makes an angle of 30 degrees with the area vector.

What is the magnetic flux through the loop?

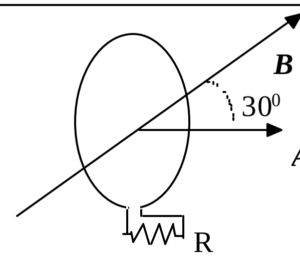


Figure 9: Magnetic Field (B) is at a 30 degree angle to the area vector.

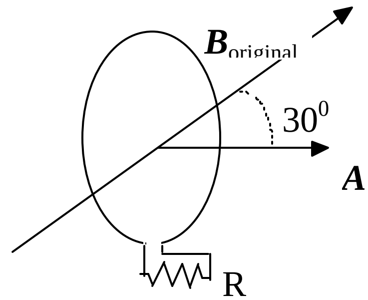
$\Phi_B =$	Weber=T m^2 (units of mag. flux)

The magnetic field *increases* from 0.30 T to 0.50 T in one tenth of a second. If we assume that it increases at a constant rate, what is that rate? (Rate = change per unit time)

$\dfrac{dB}{dt} =$

What is the induced *EMF* the loop? If the resistance in the loop is 20 Ohms, what is the induced current?

$\varepsilon =$
$I =$



> **Lenz' Law**: the direction of the induced *EMF* is such that the induced currents will produce a magnetic field which tends to counter-act the changing magnetic flux.

This means that in the previous example, since the magnetic field and therefore the flux was *increasing*, the induced current will tend to counter-act that change by creating a magnetic field in the *opposite* direction as the original field (thus subtracting from the increasing flux) Use the right hand rule to find the direction of that current, ***I***¸ and the direction of the induced magnetic field, B_{induced} created by the induced current. Sketch ***I*** and B_{induced} above.

<u>Problem 2:</u> A long solenoid (cylinder wrapped with many turns of wire) creates a magnetic field *inside* of it equal to $B = m_0 nI$ where $m_0 = 4p \times 10^{-7}\, T \cdot m / A$, n is the number of turns of wire per unit length, and I is the current in the wire. If the solenoid is very long, then outside (far from the ends) the field is zero.

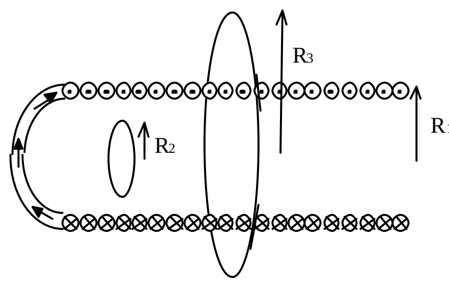


Figure 10: Large loop outside a solenoid

Consider a solenoid with 100 turns of wire per cm (n=100 cm^{-1}). If a current of 2A runs through it, what is the field inside? (Be careful of your units! You must convert turns/cm to turns/m.)

***B*=**

Let the radius of the solenoid be R_1=2 cm, the radius of a small loop inside it be R_2=1 cm and the radius of a big loop outside it be R_3=5cm. Find the flux through the two loops. (Be careful! Remember the field outside the solenoid is zero, so there is only flux through part of the area of the outer loop.)

Small Loop: Φ_B =
Large Loop: Φ_B =

Now suppose that the current is decreases at a constant rate from 2A to zero over a time of 0.2 seconds. What is the *EMF* induced in each of the two loops?

Small Loop: e_2 =
Large Loop: e_3 =

Problem 3: A circuit consists of three fixed metal rails (shown in black) on which a fourth rail (shown in white) is moving at constant speed $v_{0.}$

A constant magnetic field of magnitude *B* comes out of the paper.

The area inside the closed circuit is A=W x H. Show that $\frac{dA}{dt} = H \cdot v_0$ (*What is constant? What is changing with time?)*

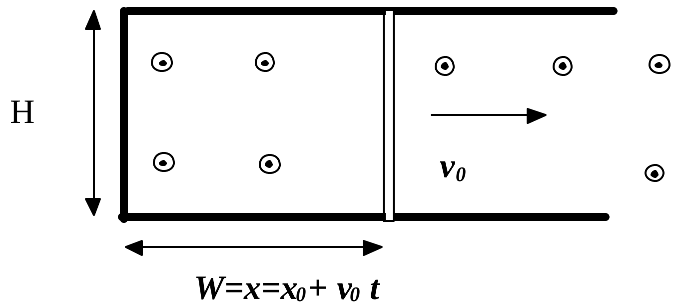


Figure 11: The areas increases as the bar travels along the rails.

If the black rails have zero resistance and the white rail has resistance *R*, show that the magnitude of the current in the rail is $I = BHv_0 / R$.

The direction of the current should be such as to counter-act the changing flux. Is the flux increasing or decreasing? Should the induced current add to the original *B* field or subtract from it? (Hint: another way to tell the direction of the induced current is that the force on the white rail, $\vec{F} = I\vec{L} \times \vec{B}$, should also be in a direction which opposes the changing flux by trying to stop the change.) Draw the direction of the current on the figure.

Problem 4: An AC (Alternating Current) generator can be made by rotating a current loop in a magnetic field. If the loop of area A rotates at angular frequency w then we can show that the flux changes (because the angle between ***A*** and ***B*** changes) so that:

$$-\frac{d\Phi_B}{dt} = -|A||B|\frac{d}{dt}\cos q = |A||B|\sin q\,\frac{dq}{dt}$$

The induced *EMF* thus varies sinusoidally with time: $e(t) = V_{\max}\sin q = V_{\max}\sin(w\,t)$

We wish to generate AC voltage with amplitude $V_{\max}$=170 Volts at a frequency of $f = w/2p$ =60Hz. If we have a magnet which can create a field of *B*=0.25 T, what area of loop will we need? What area would we need if there were 100 turns of wire in the loop?

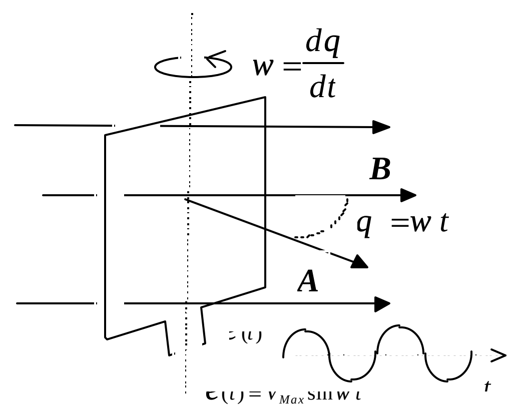


Figure 12: A loop of wire rotates in a magnetic field.

A=	A(100 turns)=

28. Electromagnetic Induction

Introduction

Electromagnetic Induction is one illustration of the fundamental link between electricity and magnetism. In 1820 Hans Oersted (during a demonstration in a physics class) noticed the deflection of the needle of a compass placed near a current carrying wire – showing that the moving electric charges in the current where creating a magnetic field. In this lab we will see that changing magnetic fields can also create potential differences which cause currents to flow. This phenomenon was first demonstrated by Michael Faraday and is known as electromagnetic induction.

Equipment

Large green power supply	Green primary and secondary coils	Red coil with 440 turns or 220 turns	compass
galvanometer (0-500 microamps)	Alligator clips Multimeter 1.5 Volt Battery	DPST switch steel and Al rods	2 bar magnets (weak and strong)

Theory (No Calculus)

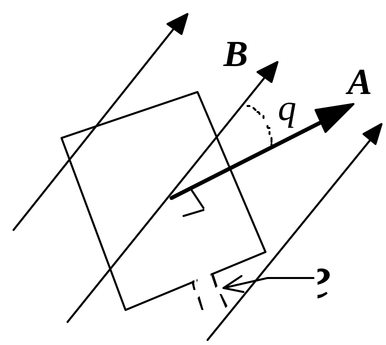


Figure 13: If the magnetic flux through any closed loop is changing as a function of time, an induced voltage will be created.

The word "flux" comes from the Latin word for "flow" and in physics refers to the "flow" of a vector quantity through an area. A current of water can be described by a vector since it has a magnitude (amount of water per time) and a direction (direction of the current). If we imaging placing a tennis racket in the water then the "flux" of water through the head of the racket would depend on 1) the strength of the current, 2) the area of the racket, and 3) the angle between the area of the racket and the direction of the current. Electric and magnetic fields are vector quantities and so we can define either an electric or a magnetic flux: the amount of electric or magnetic field passing through an area. It depends on the strength of the field, the area through which it passes and the angle between the area and the field. For a fixed area, A¸ and a magnetic field given by B, the flux of magnetic field is given by

$$\Phi_B = BA\cos q \qquad \text{(fixed, flat area and constant magnetic field)}$$

The angle, q is the angle between the area vector and the magnetic field, both of which are shown in Figure 13.

Electromagnetic Induction: Faraday's Law of Induction states that if the magnetic flux through any closed loop is changing as a function of time, then a voltage or *electromagnetic force (emf)* will be induced around that loop.

$$e = -\frac{\Delta\Phi_B}{\Delta t} \qquad \text{(changes at a constant rate)}$$

If the loop is part of a closed circuit, then the induced *emf* will create and induced current. In this lab you will look at the current induced in a coil of wire (a solenoid) as the magnetic flux inside the solenoid is changed. If the magnetic field goes down the axis of a solenoid with N coils of wire, then, when the magnetic field changes at a constant rate, the induced *emf* is given by

$$e = -NA\frac{\Delta B}{\Delta t} \qquad \text{(note: this equation is a special case as described above!)}$$

The direction of the induced voltage or current is predicted by *Lenz' Law* which is described below.

Theory (Calculus)

Magnetic Flux: In discussing Gauss' Law we have already defined electric flux. We can similarly define *magnetic flux* as the integral of the magnetic field passing through a surface. When the magnetic field is uniform over a flat surface characterized by the area vector, $\vec{A}$, the integral reduces to the scalar product of the magnetic field, $\vec{B}$, and area vector:

$$\Phi_B = \int \vec{B} \bullet d\vec{A} = \vec{B} \bullet \vec{A}$$

Electromagnetic Induction: Faraday's Law of Induction states that if the magnetic flux through any closed loop is changing as a function of time, then a voltage or *electromagnetic force (emf)* will be induced around that loop.

$$e = -\frac{d\Phi_B}{dt}$$

For the case of uniform magnetic field over a flat area, we can apply the chain rule to show that

$$-\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(\vec{A} \bullet \vec{B}\right) = -\left(|A|\cos q_{BA}\frac{dB}{dt} + |B|\cos q_{BA}\frac{dA}{dt} - |B||A|\sin q_{BA}\frac{dq_{BA}}{dt}\right)$$

Thus an induced voltage will be observed if:

the magnetic field is changing as a function of time, so the term $|A|\cos q_{BA}\frac{dB}{dt}$ is non-zero

the area is changing as a function of time, so the term $|B|\cos q_{BA}\frac{dA}{dt}$ is non-zero

the angle between the area vector and field is changing as a function of time, so the term $|B||A|\sin q_{BA}\frac{dq_{BA}}{dt}$ is non-zero.

If the loop is part of a closed circuit, then the induced voltage will create an *induced current* whose magnitude depends on the resistance in the circuit. If the flux is through a coil of *N* turns of wire, then the flux and resulting *emf* are simply multiplied by *N*.

Lenz' Law

States that the induced voltage or *emf* will be in a direction so as to *oppose* the changes in the magnetic flux. For example, if the flux is *increasing* the *emf* will be in a direction that would cause current to flow around the loop in a direction that would try to *decrease* the flux by subtracting from the existing magnetic field. If the flux is decreasing, on the other hand, the induced current would resist this change by creating a magnetic field which adds to the existing one.

Procedure

In this lab you will qualitatively examine the effects of changing magnetic field by observing currents induced in a solenoid (cylindrical coil of wire). Since we don't have meters capable of accurately measuring the rate of change of current, you will make qualitative measurements and then determine whether your observations agree with the theory of electromagnetic induction and Lenz' Law.

For Your Report Include the usual cover page and introduction. Label sections clearly (corresponding to headings in this handout). You should take the time during each part of the lab to record your observations and conclusions (discuss them with your lab partners) and write them out using complete sentences before proceeding on to the next section.. Write an overall summary for your conclusion and decide whether your observations support the theory of electromagnetic induction.

IMPORTANT SAFETY NOTES

To avoid damage to equipment be sure and observe the limitations given for current and voltage.

Use the external switch to apply current to the solenoids. Ask your instructor if you are not sure how to construct the circuit. **Do not connect or disconnect wires unless the switch is open and there is no current flowing.**

Avoid contact with wires and connectors while the switch is closed and current is flowing. **The currents used will be unpleasant to come in contact with.**

The galvanometer is very sensitive and is only used with very small currents. **DO NOT** connect the galvanometer to the circuit containing the power supply.

The direction of the Galvanometer.

The galvanometer measures very small currents with a needle which deflects right or left, depending on the direction of the current. (The needle is attached to a small coil which is attached to a spring and placed between the poles of a magnet, current flowing through the coil causes it to deflect due the torque created on the current loops by the magnetic field.) In this section you will establish how the direction of the current is related to the direction of deflection.

By briefly touching a lead from the battery to the galvanometer (in effect “closing a switch” briefly) you can find how the direction of deflection corresponds to the direction of current.

Knowing that the (positive) current flows from the positive to negative terminals of the battery, determine which direction the current must flow through the galvanometer to cause it to deflect left or right. Reverse the battery and make sure that the needle deflects in the opposite direction.

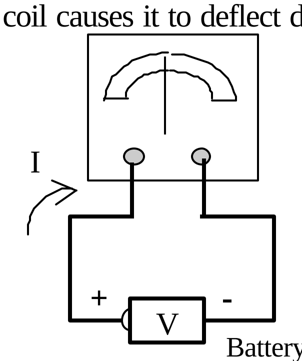


Figure 14: Circuit using a battery to test galvanometer: EXCESSIVE CURRENT WILL DAMAGE THE GALVANOMETER.

Record your observations below and in your report:

Direction of current into galvanometer *needle deflects right or left*

When the current goes in the left terminal of the galvanometer, the needle deflects ____________________

When the current goes in the right terminal of the galvanometer, the needle deflects ____________________

Magnetic Field around the bar magnets

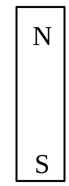


Figure 15: Sketch the field lines around the bar magnet

Lay a bar magnet flat on the table and use the compass to investigate the shape of the magnetic field around the bar magnet. Recall that the compass needle points along the magnetic field with the North end pointing in the direction of the field. The intensity can be estimated by noting how strongly the compass needle spins. In your notes you should record the direction and intensity of the field around the North pole of the magnet, near the center, and near the South pole. Make a sketch of what you suppose the magnetic field lines to look like around the bar magnet.

For this lab you will need two bar magnets of different strengths. Estimate the relative strengths of the two bar magnets (using compass and the steel rod).

Using the Bar magnets to induce current in a solenoid.

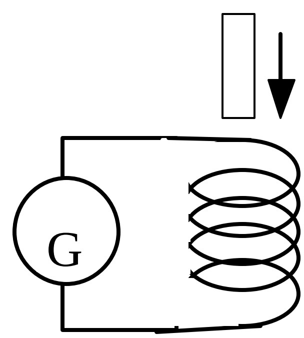


Figure 16: Using a bar magnet to induce current in a solenoid.

You should have three solenoids. One set of primary and secondary coils has green insulation. The “primary” coil fits inside the “secondary” coil The second solenoid has red insulation and actually consists of two sets of coils, one inside the other. Terminals on the end are labeled 440T for 440 turns and 220T for 220 turns and allow connection to either set of coils.

Connect the galvanometer to the red solenoid containing 440 turns of wire. (There is NO POWER SUPPLY in this circuit)

Using the stronger bar magnet, move the magnet in and out of the coil, noting and recording the effects. You should note the effect of 1) the speed with which you move the magnet and 2) changing the polarity of the magnet.

Insert the North pole as quickly as possible. Pause when it is inserted about half way. Note the galvanometer deflection and record its maximum deflection. Repeat your motion several times in order to verify your estimate of the maximum deflection.

Start with the North pole inserted halfway, then withdraw it as quickly as possible. Again record the direction and estimate the maximum deflection.

Repeat, this time moving the magnets slowly.

Repeat, for both fast and slow motion, this time using the South poles of the magnets.

Repeat using the 220 turn coil and note the differences.

Repeat using the 440 turn coil and a weaker magnet and note the differences.

Summarize your results and draw conclusions based on your observations.

Investigating the magnetic field inside a solenoid.

Inside a solenoid the magnetic field is fairly uniform. We can use Ampere’s Law to show that it is equal to $B = m_0 nI$ where $m_0 = 4p \times 10^{-7} \frac{T \cdot m}{A}$, n is the number of turns of wire per unit length and I is the current in the solenoid. Thus we can create a changing magnetic field inside a solenoid by changing the current. By placing this solenoid inside a second solenoid, we can induce currents in the second solenoid as we change the current and the flux changes with time.

With the power supply not connected to any circuit, use the multimeter to set your power supply to a reading of about 5 Volts. (On the large power supplies the voltage switch should be set to "B" and the knob turned to about "1") .

FOR THE REMAINDER OF THE LAB DO NOT INCREASE THE SETTING ON THE POWER SUPPLY. (Switch to "off" if desired but do not change the setting.) **USE THE EXTERNAL SWITCH TO TURN THE CURRENT ON AND OFF. DO NOT TOUCH METAL WIRES OR CONNECTORS WHILE SWITCHING THE CURRENT**.

WITH THE SWITCH OPEN connect the positive terminal of the power supply to the one terminal of an external switch. Connect the output side of the switch to one terminal of the small solenoid. Connect the second terminal on the small green solenoid to the negative terminal of the power supply. Examine the solenoid and determine which way current flows in it.

Lay the solenoid down on the table. Without touching any metal connections, CLOSE the switch (current flows) and investigate the magnetic field around the solenoid with a compass. Since the field is weak, it may help to switch the current on and off while looking at the compass needle to see how it deflects. You should be able to detemine the direction of the magnetic field as it goes through the solenoid.

OPEN the switch and reverse the leads. Repeat the previous step and verify that reversing the current reverses the direction of the magnetic field.

Changing current in one solenoid to induce current in a second solenoid.

OPEN THE SWITCH to stop the current in the small coil.

Place the small green coil inside the larger green solenoid.

Connect the Galvanometer to the larger solenoid. You will be changing the current in the smaller coil (the "Primary Coil") and observing the effect on the current in the larger, outside coil (the "Secondary Coil").

Close the switch, sending current through the primary coil, and observe the effect on the current in the secondary coil. Note the direction of deflection and estimate the magnitude of largest deflection. (It may help to close and open several times to get an estimate of the maximum deflection.)

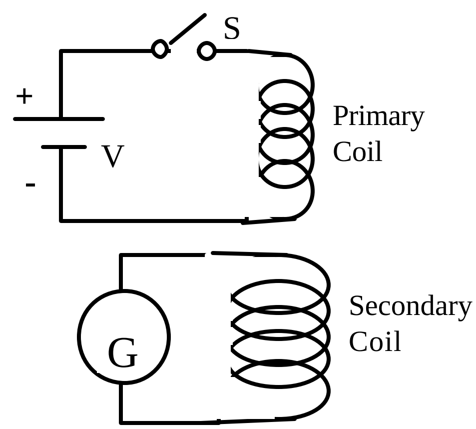


Figure 17: Changing current in the primary coil will cause a changing flux in the secondary .

Leave the switch closed for a moment. What is the deflection of the galavanometer?

Open the switch again, noting the effect on the current in the secondary coil.

Repeat as needed to verify your observations and to allow all lab partners to observe and record the results.

WITH THE SWITCH OPEN (no current), change the direction of the current in the primary coil be reversing the leads. Again close and open the switch to start and stop the current, noting the effect on the current in the secondary coil.

Summarize your results and draw conclusions.

Magnetic effects of materials.

When a magnetic field exists in a material, rather than in vacuum, the magnetic field will be changed by the magnetic properties of the material. Here you will compare the magnetic properties of Aluminum and Steel. (Note: steel is an alloy containing iron and smaller amounts of other elements.)

Try to pick up both the aluminum and steel with your bar magnet. Does it make a difference whether you use the North or the South pole of the bar magnet?

Using the same circuit as above, WITH THE SWITCH OPEN, insert the aluminum rod inside the primary coil. Open and close the switch and note any effects on the current in the secondary coil.

WITH THE SWITCH OPEN, replace the aluminum rod with the steel rod. Repeat and record your observations.

Summarize and draw conclusions about the magnetic properties of aluminum and steel.

When a magnetic field exists in a material, rather than in vacuum, the constant m_0 is replaced by a value m that depends on the material. Speculate on the values of m for aluminum and steel.

To observe an interesting effect, try placing the steel rod halfway inside the primary (do this from the side without contacts and wires). Turn the current on and see what happens.

Effect of number or turns of wire in the primary and secondary.

Continue to use the smaller green coil as the primary. With the switch open, replace the secondary (larger green) coil by the red coil with 220 turns. Close and open the switch and note the effects. With the switch open, change the connections so that the 440 turn coil is used as the secondary. Close and open the switch and note the effects. With the switch open, move the connections from the small green coil to the leads for the 220 turn red coil so that it becomes the primary (while the 440 turn coil remains the secondary). Close and open the switch and note the effects.

Summarize your results and draw conclusions about the effect of the number of turns in the primary or secondary coils.

Conclusions

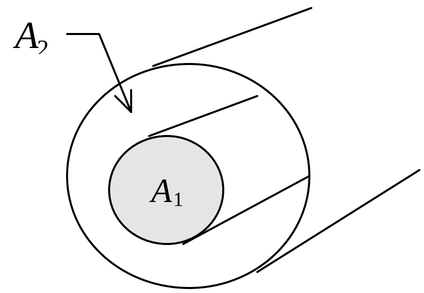


Figure 18: Two concentric solenoids.

Why does the direction of the induced current depend on which pole of the permanent magnet is inserted into the solenoid?

Why does the magnitude of the induced current depend on the strength of the bar magnet?

The magnetic field inside a solenoid is given by: $B = m_0 nI$. Outside the solenoid the field may be approximated as zero. Suppose that a solenoid of area A_1 and turns per unit length n_1 is placed inside a second solenoid of area A_2 , turns per unit length n_2 and length L_2 (total number of turns in second coil $N_2 = L_2 n_2$). Explain why the *emf*, e , induced in the *second* coil by a changing current, $\frac{dI_1}{dt}$ in the *first* coil is given by $e = -N_2 A_2 m_0 n_1 \frac{dI_1}{dt}$ If the change is at a constant rate, then $\frac{dI_1}{dt}$ may be replaced by $\frac{\Delta I_1}{\Delta t}$)

If the secondary solenoid is in a circuit with resistance R, there will be an induced current, $I_2 = e/R$. Discuss how the equations predict the direction of the current in the secondary coil depends on whether the current in the *primary* coil, I_1, is increasing or decreasing. Does this match your observations? (Hint: consider the effect of the sign of $\frac{dI_1}{dt}$ -- is it increasing or decreasing?)

Given the expression above, how do the equations predict the induced current depend on the number of turns in the primary? In the secondary? Does this match your observations?

Using the Bar magnets to induce current in a solenoid.

440 Turn Secondary 220 Turn Secondary

Magnet 1 :

Strength of magnet =

North Pole	Maximum Deflection, *mA*	Direction of Current	Maximum Deflection, *mA*	Direction of Current
Moves quickly inward				
Stationary inside				
Moves quickly outward				
Moves slowly inward				
Moves slowly outward				
South Pole	Maximum Deflection, *mA*	Direction of Current	Maximum Deflection, *mA*	Direction of Current
Moves quickly inward				
Stationary inside				
Moves quickly outward				
Moves slowly inward				
Moves slowly outward				

Magnet 2 :

Strength of magnet =

North Pole	Maximum Deflection, *mA*	Direction of Current	Maximum Deflection, *mA*	Direction of Current
Moves quickly inward				
Stationary inside				
Moves quickly outward				
Moves slowly inward				
Moves slowly outward				
South Pole	Maximum Deflection, *mA*	Direction of Current	Maximum Deflection, *mA*	Direction of Current
Moves quickly inward				
Stationary inside				
Moves quickly outward				
Moves slowly inward				
Moves slowly outward				

Changing current in one solenoid to induce current in a second.

Primary=small green coil Secondary=large green coil

	Initial Current Direction		Opposite Current Direction	
	Maximum Deflection, *mA*	Direction of Current.	Maximum Deflection, *mA*	Direction of Current.
While closing switch				
Current constant				
While opening switch				

Magnetic effect of materials

Primary=small green coil Secondary=large green coil

	Aluminum Rod		Steel Rod	
	Maximum Deflection, *mA*	Direction of Current.	Maximum Deflection, *mA*	Direction of Current.
While closing switch				
Current constant				
While opening switch				

Effect of number of turns in primary and secondary

Primary=small green coil Secondary = red coil

	Secondary=220T		Secondary=440T	
	Maximum Deflection, *mA*	Direction of Current.	Maximum Deflection, *mA*	Direction of Current.
While closing switch				
Current constant				
While opening switch				

	Primary=220T Secondary=440T		Primary=440T Secondary=220T	
	Maximum Deflection, *mA*	Direction of Current.	Maximum Deflection, *mA*	Direction of Current.
While closing switch				
Current constant				
While opening switch				

29. AC Circuits: Driven Oscillations and Resonance

Introduction

In an AC (alternating current) circuit the direction of the current in the circuit alternates as a function of time. Since the current is time dependent, both capacitors and inductors play important roles in AC circuits. The response of an AC circuit will also depend on the driving frequency of the AC power supply.

Equipment

Function generator	Capacitance decade box (0.1nF-100nF)	Resistance decade box (1-99,999Ω)	Inductances (30mH &100mH)
Alligator clips and banana plug leads	Oscilloscope	Scope probes (3 sets, BNC to alligator or banana)	Multimeter Graph paper

Theory

Some important AC circuit elements are:

Capacitors:

A capacitor stores charge +Q and -Q on either plate. The amount of charge, Q, is proportional to the voltage across the capacitor. Potential energy is stored in the electric field between the plates. Because the voltage depends on charge rather than current, the voltage in the capacitor is out of phase with the current (voltage V_C lags current by $p/2$ when there is no resistance).

Inductors:

Changing current results in an induced *emf* or voltage that is proportional to, and opposes, the rate of change of current. Potential energy is stored in the magnetic field created by the current. Because the voltage depends on the derivative of current, the voltage in the inductor is out of phase with the current (voltage V_L leads current by $p/2$ when there is no resistance).

Resistors

A resistor has voltage proportional to current (Ohm's Law: $V_R=IR$) Energy is dissipated in the resistor. The voltage on a resistor, V_R, is in phase with the current

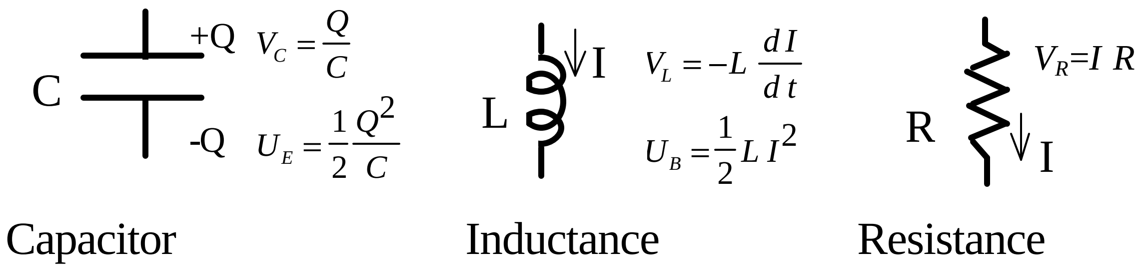


Figure 19: AC circuit elements. Voltage and energy in a capacitor depends on charge, which changes as current flows. Voltage and energy in an inductor depend on changes in the current. The resistor is a passive circuit element: voltage is directly proportional to current, and energy is dissipated as heat.

<u>The AC circuit: Driven oscillations and resonance.</u>

An AC circuit driven by a sinusoidally varying power supply has a sinusoidally varying current and voltage. In the different circuit elements, current is not always "in phase" with voltage. That is, current may reach its maximum value at a different time than voltage reaches its maximum value.

Figure 20 shows how the voltage and current can vary in an AC circuit. The magnitude of the current and the phase difference between the voltage and current depend on circuit elements such as resistors, capacitors and inductors.

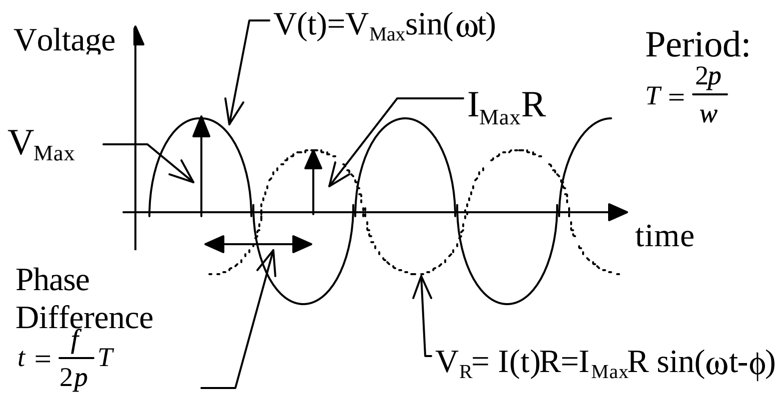


Figure 20: AC circuit. Solid line shows voltage from AC power supply, Dashed line shows voltage in resistor, which is in phase with the current. The voltage and current peak at different times. The voltage leads the current here.

The relationship between peak voltage and peak current is characterized by the *impedance, Z.*

$$V_{Max} = I_{Max} \cdot Z$$

If the circuit is driven by a power supply with peak voltage V_{max} and angular driving frequency w_d then

$$V(t) = V_{Max}\sin\left(w_d \cdot t\right) \qquad \text{and} \qquad I(t) = I_{Max}\sin\left(w_d \cdot t - f\right)$$

The smaller the impedance, the larger the current. The impedance, *Z*, and the phase depend not only on the resistance, capacitance and inductance in the circuit, but also on the driving frequency.

Resonance in the Series RLC circuit

A series RLC circuit is shown in Figure 21. For this circuit:

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} \qquad \text{and} \qquad \tan f = \frac{\left(X_L - X_C\right)}{R}$$

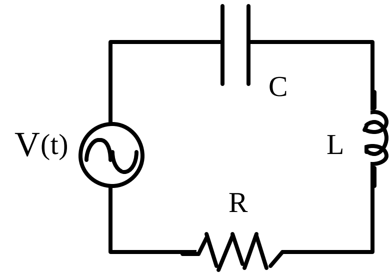


Figure 21: Series RLC circuit

where

$R =$ Resistance

$X_L =$ Inductive Reactance $= w_d \cdot L$

$X_C =$ Capacitive Reactance $= \dfrac{1}{w_d \cdot \mathrm{C}}$

The peak current in the circuit depends on the frequency of the driving voltage. For small frequencies the capacitor has a large reactance and for large frequencies the inductive reactance becomes large. The peak current is given by

$$I_{Max} = \frac{V_{Max}}{Z} = \frac{V_{Max}}{\sqrt{R^2 + \left(X_L - X_C\right)^2}}$$

so the largest value of peak current can be achieved when $X_L = X_C$ which is when $w_d \cdot L = 1/\left(w_d \cdot C\right)$. Notice that this occurs when the driving frequency w_d is equal to the “natural” oscillating frequency of the series LC circuit, $w_r = 1/\sqrt{LC}$. This situation is known as *Resonance* and the frequency $w_r = 1/\sqrt{LC}$ is called the resonant frequency of the circuit. If the resistance is small, the peak voltage can become quite large. In fact, in the limit that the resistance is zero, the peak current would theoretically be infinite. In reality the power supply would limit the current.

Procedure

Setting up the oscilloscope

You will begin by setting up the oscilloscope to detect the sinusoidal voltage output by the function generator. Connect Channel 1 of your scope to the sine wave output. All inputs should be set to AC and the trigger should be set to Channel 1. On the function generator set the frequency to midrange on the 10 kHz scale and the amplitude to midrange. Set the time scale of your scope so that you can observe several periods of oscillation. Make sure all knobs on the scope are “clicked” into their calibrated positions. Adjust the position, intensity and trigger of your scope until you can see the voltage output from the function generator. Ask for help if you are unsure. Use the position adjustments on your scope to verify which trace is CH 1 and which is CH 2. Vary the amplitude, frequency, and oscilloscope controls until you are sure you understand what you are seeing. Sketch what you see (including axis and division marks) and record the Volts/Division and Time/Division settings from the scope. Label V_{max} and the period, T. Calculate the frequency, f = *1/T,* and angular frequency, $w_d = 2\pi f$. Change the frequency by a factor of ten and re-adjust the scope. Measure the period and calculate the frequencies.

<u>A capacitive circuit:</u>

Set up the circuit shown in Figure 22 using the resistance and capacitance decade boxes. The “MF” on the capacitance decade box represents microfarads (μF). Use the following settings:

R about 500 Ω	C about 0.1 μF	*f* about 500 Hz

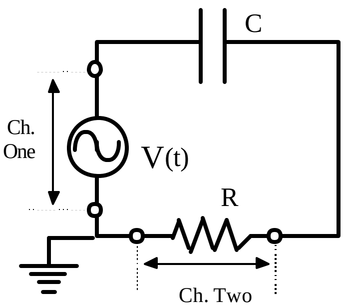


Figure 22: RC Circuit

Connect Ch. 1 of the scope across the voltage source and Ch. 2 across the resistor. *Note that Ch. 1 and Ch. 2 actually have the same ground which must be connected to the same “ground” in the circuit.* Since the voltage on the resistor is $V_R = IR$, Ch. 2 will provide an indirect measure of the current in the circuit: $I = V_R/R$.

Adjust your scope until you can see both Voltage (Ch. 1) and Current (Ch. 2). Adjust the Volts/div Time/div on the scope so that Voltage and Current both display well. Make a sketch of what you see, including axis and division markings on the data sheet. Record the Volt/div for both channels and the Time/Div., and measure the maximum voltages and the period and calculate the maximum current and frequencies. On your sketch label amplitudes, V_{Max} and $V_R=I_{Max}R$, the period, T, and the time corresponding to the phase difference, $t = \left(f/2p\right)T$. (See Figure 20). From the amplitudes for V_{Max} and V_R determine the impedance, *Z*.

Does the voltage lead the current (peak before the current) or lag the current (peak after the current)? Calculate theoretical values for the impedance and phase and compare to experimental values.

Vary the a) frequency and b) the capacitance C (or inductance L for the next section) and observe the effects on the amplitude of the current and the phase. Discuss whether or not the behavior is what you would expect from the theoretical equations for impedance and phase.

<u>An inductive circuit:</u>

Set up the circuit shown in Figure 23 using the resistance decade box and L=100mH inductance. Use the following settings

R about 100 Ω	L about 100 mH	*f* about 50 kHz

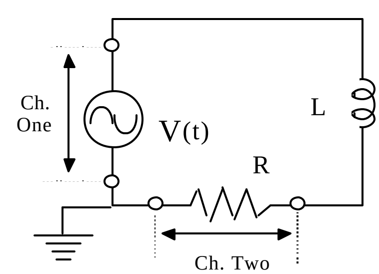


Figure 23: An LR circuit

Repeat the measurements and observations you made for the RC circuit (see data sheet).

Series RLC circuit and resonance

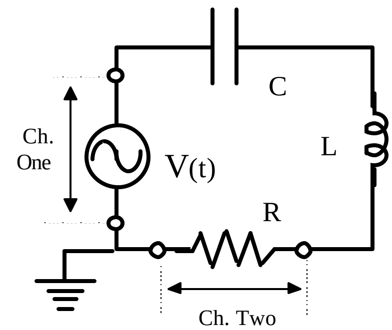


Figure 24: Series RLC circuit

Set up the series circuit shown in Figure 24 using C = 0.1 μF and L = 100 mH. Set the resistance to about 1000 Ohms. Observe the current (i.e. V_R) in the circuit as you vary the frequency. Find the frequency that results in the greatest current (i.e. greatest V_R). This is the resonant frequency, f_r, for the circuit. Measure the amplitudes, period and phase. Calculate experimental values for the impedance and phase at the resonant frequency.

Repeat, substituting the 30mH impedance. Make a prediction for the new resonant frequency and then find it experimentally. At the resonant frequency measure the amplitudes, period and phase and compare the impedance and phase to the theoretical values.

Which way (earlier time or later time) does the peak current move when the driving frequency is less than the resonant frequency. When it is greater than the resonant frequency?

Width of the resonance

Change the resistance to about 100 Ohms. Vary the frequency around resonance and observe and differences in comparison to the circuit with 1000 Ohms resistance. Notice any changes in the "width" of the resonance (i.e. range of frequencies over which V_R decreases to 1/2 of its resonance value).

Questions

Compare the capacitive circuit to the inductive circuit. For each, discuss how the impedance and phase changed with frequency. Discuss both your experimental observations (of amplitude and phase) and what the theory would predict.

For the RLC circuit, which way (earlier time or later time) does the peak current move when the driving frequency is less than the resonant frequency. When it is greater than the resonant frequency? Is this what you expect? Is the capacitor or the inductor more important at high and at low frequency.

Is the width of the resonance greater or narrower (i.e. "sharper") with 100 Ohms than with 1000 Ohms?

Capacitive (RC) circuit

Sketch:

Oscilliscope Settings:
Time/Div:

Ch. 1: Volts/Div

Ch. 2: Volts/Div

R= C=

Experimental (from oscilliscope trace) -- *include units*

T=	f =	w =
V_{Max}=		
V_R =	$I_{Max}=V_R/R$	Z= V_{Max}/I_{Max}=
t (for phase difference)=		$f = 2p \cdot (t/T) =$

Theoretical Values (give formula)

Z=	% Diff.
f =	% Diff.

Does the voltage lag or lead the current? How does the current amplitude and phase change with frequency?

Inductive (RL) circuit

Sketch:

Oscilliscope Settings:
Time/Div:

Ch. 1: Volts/Div

Ch. 2: Volts/Div

R= L=

Experimental (from oscilliscope trace) -- *include units*

T=	f =	w =
V_{Max}=		
V_R =	$I_{Max}=V_R/R$	$Z= V_{Max}/ I_{Max}$=
t (for phase difference)=		$f = 2p \cdot (t/T) =$

Theoretical Values (give formula)

Z=	% Diff.
f =	% Diff.

Does the voltage lag or lead the current? How does the current amplitude and phase change with frequency?

Series RLC circuit

R= L= C=

Experimental (from oscilliscope trace) -- *include units*

T=	f =	w_d=
V_{Max}=		
V_R =	$I_{Max}=V_R/R$	$Z= V_{Max}/ I_{Max}$=
t (for phase difference)=		$f = 2p \cdot (t/T) =$

Theoretical Values (give formula)

w_r=	% Diff.
Z=	% Diff.
f =	% Diff.

Series RLC circuit

R= L= C=

Experimental (from oscilliscope trace) -- *include units*

T=	f =	w_d=
V_{Max}=		
V_R =	$I_{Max}=V_R/R$	$Z= V_{Max}/ I_{Max}$=
t (for phase difference)=		$f = 2p \cdot (t/T) =$

Theoretical Values (give formula)

w_r=	% Diff.
Z=	% Diff.
f =	% Diff.

Chapter 6: Oscillations and Waves

30. Hooke's Law and Simple Harmonic Motion

Introduction

The force applied by an ideal spring is governed by *Hooke's Law*: F=-kx. Because the force is proportional to displacement of the spring, a mass attached to the spring will undergo *simple harmonic motion*. In this lab we will verify Hooke's Law and learn about simple harmonic motion. The motion of the spring will be compared to motion of a pendulum.

Equipment

Spring	Rods, Clamp	Mass set	String
Tape	Meter Stick	Mass hanger	pendulum Bob
Motion Detector		ULI interface	Graphing, MacMotion software

Theory

Hooke's Law

The first part of this experiment is designed to verify Hooke's law. Hooke's law states that the force F applied to a mass by a connected spring is proportional to the displacement, $(x\text{-}x_0)$of the mass from its equilibrium position, that is,

$$F = -k \cdot (x - x_0)$$

where the proportionality k is the spring constant. The minus sign occurs because the force is opposite to the direction of displacement. Hooke's law can be used to measure the spring constant, k.

Simple Harmonic Motion

The second part of the experiment verifies that the motion under this force is "simple harmonic motion" with a period, *T*, of

$$T = 2p \cdot \sqrt{\frac{m}{k}}$$

Thus, *k* can also be measured by measuring the period of the mass' oscillation.

Oscillation of a pendulum

In the final part of the experiment we determine that a simple pendulum (mass hanging from a string) also appears to exhibit simple harmonic motion and test the dependence of the period of a pendulum on the length of the pendulum.

Procedure

<u>Part 1: Verifying Hooke's Law and measuring the spring constant</u>

In this part, the ULI with the motion detector is used to measure the displacement of "blocks" of different masses. If x_0 is the equilibrium position of the spring (no mass attached), then the displacement is $(x\text{-}x_0)$, where "x" is the position measured by the detector (see Figure 1). According to Hooke's Law, the force applied by the spring is:

$$F(x) = -k \cdot (x - x_0) = -k \cdot x + k \cdot x_0$$

Graphing software, such as Kaleidagraph, is used to make a plot of F versus x. Hooke's law is verified if F vs. x is a linear line. The slope of this line represents the spring constant k (the intercept will be kx_0).

Data collection

DO NOT DROP THE MASSES ON THE MOTION DETECTOR - tape in place

Set up a spring-mass system as shown in Figure 1. (The spring can hang off the table and the detector may sit on the floor.) In this part you will use the motion detector to measure distances for stationary masses.

Connect the interface board to the computer (modem port). Connect the motion detector to port 2 of the interface board. Turn on the power to the interface board.

Open MacMotion. Clicking **Start** and **Stop** will start and stop the motion detector. Letting the software run while the block is stationary allows you to get a reading on the distance for different masses.

Measure the distance of the hanger with the maximum weight first. The distance between the object to the motion detector must always be greater than about 50 cm.

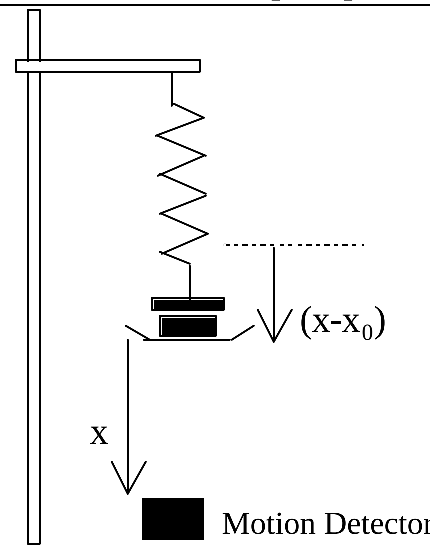


Figure 1: Mass on spring with motion detector.

Click **Start** to collect data. The distance as a function of time should be a horizontal line. Click **Stop** to stop the motion detector.

Select **Stats...**under the **Analyze** menu. Record the mean value to Table 1.

Repeat the measurement of distance for different masses by placing the listed masses (Table 1) on the hanger, and recording the mean position values.

Express F in terms of Newtons and fill out the last column in Table 1.

Input F and x in Table 1 into a graphing program such as Kaleidagraph (See Appendix). Make a plot of F vs x. F vs.x is expected to be linear with slope corresponding to the spring constant k. Determine the slope by using a linear fit. If your plot does not seem to fit a straight line it may be because your masses are too close or to far from the detector at one end or the other.

Part 2: The oscillating mass

This part is designed to verify that the motion of the oscillating mass on the spring is simple harmonic motion with a period of

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad \text{(Equation 1)}$$

Because of the mass contribution from the spring, m in this equation consists of the mass of the weights m_W and the effective mass of the spring M_e . The above equation can be written as

$$T = 2\pi \sqrt{\frac{m_W + M_e}{k}}\ , \qquad \text{(Equation 2)}$$

or, by squaring both sides:

$$T^2 = \frac{4\pi^2}{k} m_W + \frac{4\pi^2}{k} M_e \qquad \text{(Equation 3)}$$

Equation (3) suggests that T^2 versus m_W is linear with slope equal to $\frac{4\pi^2}{k}$ and intercept equal to $\frac{4\pi^2}{k} M_e$. Thus both the spring constant k and the effective mass of the spring, M_e can be derived from the T^2 versus m_W line. The k derived in this method should be very close to the one obtained by the Hooke's law. The ratio of M_e to the actual mass of the spring M_s is always less than one.

Data collection

The same set-up is used in this part.

Add 100 grams to the hanger.

Give the block a small initial displacement and let it oscillate.

Set the data collection rate to its highest value. Click **Start** to collect data. The position as a function of time should look like a sine function. (That is precisely what is meant by simple harmonic motion!) Click **Stop** to stop the motion detector. Change the scale of your vertical axis (by clicking on the top and bottom numbers) until you see your graph well. If your data does not look like a nice sine wave adjust the data collection rate or the alignment of your apparatus.

Select the **Analyzer** item under the **Analyze** menu. You will use this to measure time.

The period is measured more accurately if several periods are counted, rather than just one. Between some number, n, periods, measure the peak to peak time, T_1 and T_2, and the number of the period between T_1 and T_2, n. Record them in Table 2.

Repeat steps 1-5 with different weights listed in Table 2, and record times to Table 2.

Calculate T and T^2 in Table 2.

Input T^2 and m_W(don't forget to convert Grams into Kilograms) to Kaleidagraph.

Make a plot of T^2 versus m_W.

Calculate the spring constant k and compare it with the one obtained in Part 1.

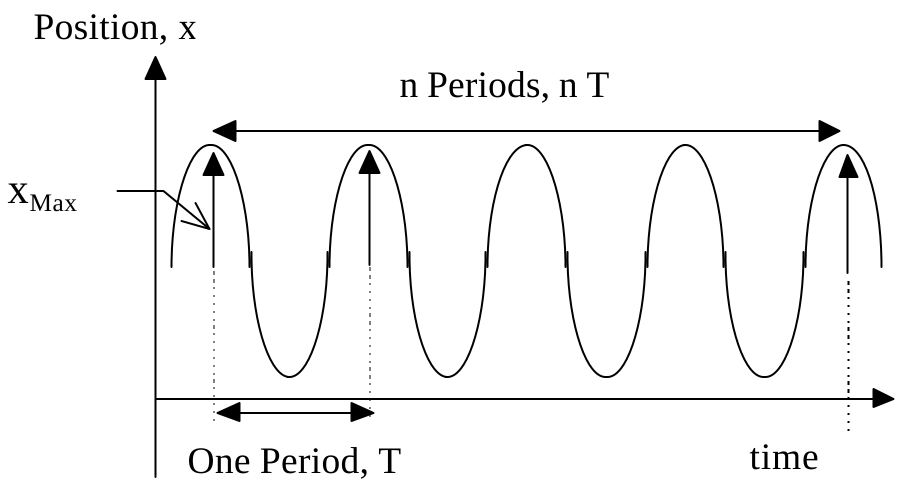


Figure 2: Simple harmonic motion: the mass oscillates like a sin function: $x(t) = x_{Max} \sin\left(\frac{2p}{T} t\right)$

Part 3: Oscillation of a pendulum

In this part, we verify the expression for the period of a pendulum. For *small angles* of oscillation, the torque on the pendulum Bob is proportional to the angle of displacement. This means that the pendulum should also undergo simple harmonic motion. We shall measure the period of oscillation and find out how it is related to the length of the pendulum: that is the distance from the axis of rotation to the center of mass of the pendulum bob.

Data Collection:

Set up a simple pendulum and position the ultrasonic motion detector to monitor the motion.
Measure the period of the pendulum for several different string lengths (Table 3)
Determine the relationship between the period of the pendulum and its length by plotting period, T, versus length or T^2 versus length
From your instructor you can find the actual relationship between period and length, which is valid for small angles of oscillation

Additional Activities: Damped Oscillations

If a large piece of cardboard with a slot in it is placed under the mass in the mass/spring setup, the air resistance experienced by the mass as it oscillates will be greatly increased. You will be able to investigate how the damping force acts to decrease the amplitude of the oscillations over time.

Table 1: Verifying Hooke's Law

M (weights on the hanger)	Position, x	For a stationary mass, F (= Mg) $g=9.8 m/s^2$
0		
100 g=0.100 kg		
200 g=0.200 kg		
300 g=0.300 kg		
400 g=0.400 kg		
500 g-0.500 kg		

Spring constant from method 1 = ______________________________

Table 2: Oscillation of the mass

m_w (grams)*	T_1	T_2	n	$T = (T_2 - T_1)/n$	T^2
50 + 100					
50 + 200					
50 + 300					
50 + 400					

*where 50 grams is the mass of the weight holder.

Spring constant from method 2 = _____________________________

Discussion: (continue on separate pages as needed)

Table 3: Oscillation of the pendulum

Length (meters)	T_1	T_2	n	T $(T_2 - T_1)/n$	T^2
0.2					
0.4					
0.6					
0.8					
1.0					

Discussion (continue on separate pages as needed):

31. Waves: Strings and Sound

Introduction

A mechanical wave is a disturbance which moves through some medium. In this lab you will study two types of waves: transverse waves on a string and longitudinal sound waves.

Equipment:

Strings: Vibrating string driver, string, tape, rods,clamps, pulley, mass hanger, weight set, meter stick, high precision scale.
Sound: Knudt's tube apparatus, cork dust, chamois cloth, rosen, meter stick, ear protection.

Theory

Waves:

Common characteristics of waves are
Wavelength: l , the minimum distance for the pattern of the wave to be repeated.
Frequency: f , the times per second the wave motion is repeated (measured in Hertz=Hz=1/sec).
Period: T, The time required to make one oscillation or for the wave to travel one wavelength.
Speed: v, the velocity of propogation of a traveling wave: $v = l \cdot f$
Amplitude: A, The maximum size of the disturbance.
A *Node* in a wave is a position where the size of the disturbance is zero. An *anti-node* is the position of maximum disturbance.

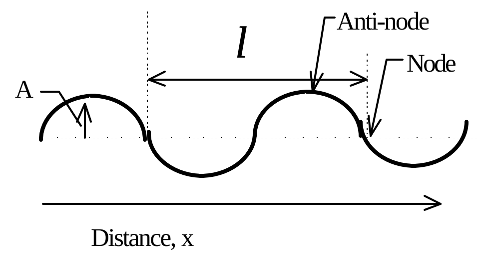


Figure 3: A standing wave oscillates in place at frequency, f. A traveling wave will propogate at speed, v. At any one point the disturbance oscillates with frequency f such that $v = l \cdot f$

Waves on a string:

A wave on a string is an example of a *transverse* wave: the disturbance in the string (displacement from a straight line) is transverse (perpendicular) to the direction of the string. The wave may be *traveling* down the length of the string or be a *standing wave* which oscillates in place. Waves on a string have characteristic frequencies which depend on the density of the string (mass per unit length, m) and the tension (t) in the string. The speed of the wave in the string is given by $v = \sqrt{t/m}$. Even though standing waves do not actually travel along the string, their wavelength and frequency are still characterized by $l \cdot f = \sqrt{t/m}$.

Sound Waves:

Sound waves in a gas are created by a mechanical disturbance which sends a wave of high and low pressure traveling through the molecules in the gas. In a solid, sound propagates as a disturbance in the positions of the atoms in the solid. Sound waves in the air are an example of a *longitudinal* wave because the changes in pressure and displacement of the air molecules are in the same direction as the wave. Each gas has a characteristic speed of sound which depends on pressure and temperature. In air the speed of sound is about v_a=331.5m/sec+0.61·T(^{0}C), where T is temperature in degrees Celsius.

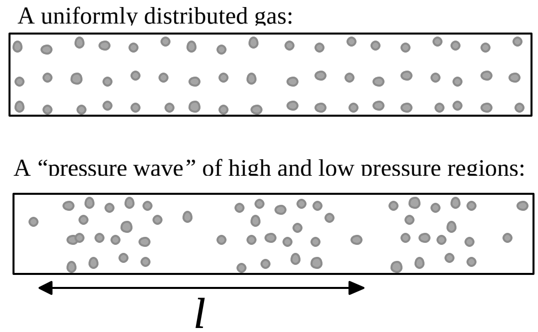


Figure 4: Sound Waves

Resonance:

Resonance occurs when the driving frequency creating a wave matches a natural frequency of the mechanical system or medium in which the wave propagates. In a string with both ends fixed, standing waves can be supported only if there is an integer number of half wavelengths along the string ($L = n \cdot l/2$): in this case the ends of the string are at nodes which remain fixed. The characteristic, or resonant frequencies of the string are then determined from the relationship $l \cdot f = \sqrt{t/m}$. In a tube or pipe with two closed ends, the gas at the end of the pipe does not move: again there must be an integer number of half wavelengths inside the pipe for it to support standing waves of sound. The characteristic frequencies are determined by the length of the pipe and the speed of sound inside the pipe. The lowest characteristic frequency is called the *fundamental* frequency and higher frequencies are called *overtones* or *higher harmonics*.

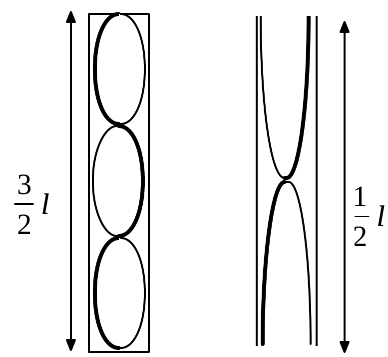


Figure 5: A closed pipe supports standing waves of sound with nodes at both ends. An open-ended pipe has an anti-node at the open ends.

Procedure

Waves on a String:

A string will be attached to a vibrator which will provide a driving force to excite standing waves in the string. The vibrator uses a electromagnet which alternately attracts and repels a steel reed. The magnet operates using the AC line voltage and its driving frequency is the line frequency, or 60 Hz.

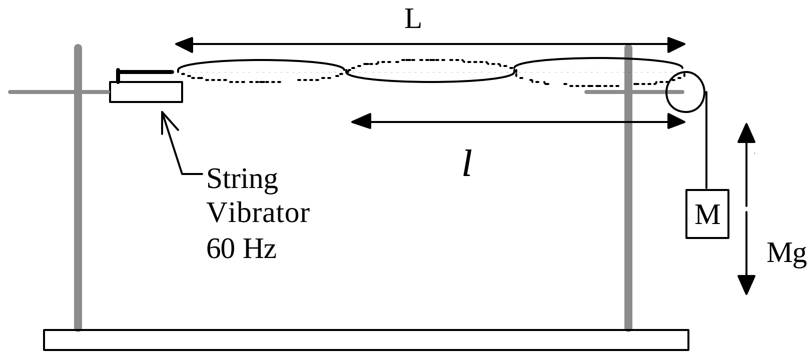


Figure 6: Standing Waves onVibrating String

Measure and record the mass of the string and its total length.

Attach the string to the vibrator and to a mass suspended from a pulley which will apply a tension, $t = Mg$, to the string. (You can loop the string around the back of the vibrating reed and use tape to fix one end to the front of the reed). Let the length of the string be about 2 meters. Record the length of the string between its fixed ends.

Once string is positioned, you can vary the tension in the string to produce standing waves in the string. The velocity of traveling waves in the string would be $v = \sqrt{t/m}$, where t =Tension and m =mass/Length. Standing waves between fixed end-points must have a half-integer number of wavelengths between the ends, $L = n\,l/2$, where n is an integer. If the length and frequency are fixed, the tension in the string can be varied to excite standing waves of different wavelengths. When the driving frequency (*f*) and a wavelength l fit the relationship $f \cdot l = v = \sqrt{t/m}$ then a resonant standing wave is excited in the string. (In a piano the length and tension of the piano wires are adjusted until the resonant frequency matches a desired note. Here the frequency is fixed by the driving force).

Experiment by applying tension to the string (pull down on the mass hanger) and observe how different harmonics can be excited as the tension varies. . Have one lab partner hold a contrasting paper behind the string if it helps you see better. <u>Record your observations</u> How does the wavelength vary with tension?

With tension adjusted so a number of half-wavelengths are excited (add or subtract masses to hold the tension) move your finger along the string and observe what happens to the standing waves. <u>Record your observations</u>. What happens if your finger in on a node? On an anti-node?

Since the vibrating end is not rigidly fixed, it is not a true node. The true wavelength can be measured by measuring the distance between nodes (which is $l/2$) You can measure from the fixed end (above the pulley) to the last node before the vibrating reed and divide by the number of loops to get the length of one half wavelength.

Now measure the effect more systematically. Hang different masses from the hanger and record the values of mass for which the amplitudes of different standing waves become large. For each such situation, record the number of half-wavelengths and measure the wavelength as described above.

Show that $l = \frac{1}{f\sqrt{m}}\sqrt{t} = \frac{\sqrt{g}}{f\sqrt{m}}\sqrt{M} = C\sqrt{M}$, where C is a constant. In your report explain why C is a constant for this experiment.

Use the computer (Kalaidagraph or other program) to plot wavelength versus the square root of mass and determine whether the relationship in fact fits a straight line. Discuss your results.

Find the slope of the line (using the computer) and determine the agreement with theory: *(You will have to make sure your units are the same when you make your comparison,)*

Sound Waves

SAFETY NOTES: Do not hold the glass tube while stroking the rod. Ear protectors may be desired as this lab makes an annoying , high pitched sound.

Resonant sound waves in a cavity also have wavelengths that match the dimensions of the cavity. A complex shape such as a violin has a variety of resonant frequencies. A simple shape such as a tube has resonant frequencies determined by its length. Standing longitudinal pressure waves of sound will be supported by a closed or open ended tube (See Figure 5) if a half-number of wavelengths fit within the length of the tube. Sound also travels within solids as a wave of displacement of the atoms within the solid. A metal rod will have standing-wave resonances that are like an open-ended pipe: there is an anti-node at each end and a node in the center where it is clamped. In this experiment you will excite sound waves in a metal rod which will then be transferred to the air within a glass tube. The wavelength and velocity change as the waves travel from one medium to the other, but the frequency remains the same. The pressure wave of sound within the tube will displace cork-dust which will allow you to record the wavelength of the standing wave in the glass tube.

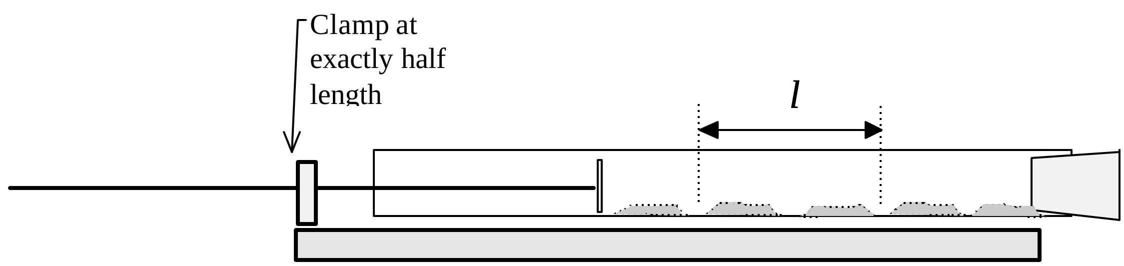


Figure 7: Kundt's Tube Apparatus

Set up the apparatus as shown, making sure that the rod is clamped precisely at its center. In a solid (unlike a gas) both longitudinal and transverse sound waves may be excited. When sound waves are excited in the rod they will have an anti-node at each end and a node at the center where it is clamped Remove the glass tube, cover the open end with your hand and shake it to distribute the cork dust evenly before replacing it. The disc at the end of the rod *should not* touch the glass tube when it is in position.

By stroking the rod along its length you can excite high frequency sound. Use a chamois cloth to stroke the rod.

IMPORTANT: Do not let your hand run off the end of the rod -- this will cause sideways mechanical oscillations which might break the glass tube. Do not hold the glass tube while stroking the rod.

The technique take some practice but when you excite the high frequency sounds you should see nodes and anti-nodes in the cork dust. The cork dust will be blown out of high pressure regions and collect in low pressure regions. If multiple half wavelengths in the glass tube does not appear to match its length then you may need to adjust the length of the glass tube (and redistribute the cork dust again) so that large amplitude resonant waves can be excited.

When you have the apparatus working well, excite a number of vibrations in the rod to get a good pattern in the cork dust. Ignoring the first and last piles of cork dust, measure the length of the piles of cork dust and count the number of nodes. Each pile of cork dust represents one half-wavelength. Calculate the wavelength of the sound in the glass tube. The velocity of sound in air is about v_a=331.5·m/sec+0.61·T(^{0}C). From the velocity and wavelength calculate the frequency of the sound in air.

The frequency of the sound will be the same in the metal. The wavelength will be such that an integer number of half-wavelengths fit within the rod length. Measure the length of the rod. Velocities of sound in various metals are: Aluminum=5104 m/sec, Brass=3500 m/sec, Steel=5000 m/sec. The exact velocity varies with temperature. From the velocity and frequency determine the wavelength of the sound in the rod. The rod is like an open-ended pipe: there should be an anti-node at each end and a node in the center where it is clamped. Compare the calculated wavelength to the length of the rod and find out which resonant wavelength best fits the data. Sketch the rod and show how the wavelengths fit within the rod.

Conclusions:

Compare the two experiments and discuss the nature of the waves in both cases.

Waves in a string:

Mass of string (m):	
Total Length (L_{total}):	Mass Density $m =$
Length between fixed ends (L_S):	

Number of loops observed n_{Total}	Suspended Mass, M	Measured length for N loops (L_N)	Wavelength $l = 2(L_N/N)$	$\sqrt{M}$

Include graph with axis labeled including units and analysis shown.

Slope of graph:	
Theoretical Prediction: $C = \dfrac{\sqrt{g}}{f\sqrt{m}}$	
Percent Discrepancy:	

Kundt's Tube (Sound): Type of metal rod used:

	Formula	Result
Temperature of Room (T):		
Velocity of Sound in Air (v_A):		
Length of dust piles Measured (L_D):		
Number of dust piles in Length (N_D)		
Wavelength in Air (l_A):		
Frequency of Sound in Air (f_A):		
Speed of Sound in Metal (v_M):		
Wavelength of Sound in Metal (l_M):		
Length of Rod (L_M):		
Number of Half-wavelengths in rod (N_M):		

Sketch of rod with wavelengths:

32. Speed of Sound and Resonance

Purpose:

The purpose of this lab is to demonstrate resonance of sound waves and determine the speed of sound traveling in air and in a metal rod.

Equipment

Resonance tube (graduated cylinder and inner glass tube), water, tuning forks, meter stick, Knundt's tube apparatus.

Procedure:

Part I: Velocity of sound in air

1) Fill the 1 liter graduated cylinder to within two inches from the top with water. Place the inner tube inside the graduated cylinder.

2) Choose a tuning fork and record its frequency.

f =________________ Hz

Strike the tuning fork with hard rubber and hold the tuning fork horizontally, with its tines one above the other about 1 cm above the open end of the inner tube. Move both the inner tube and the fork up and down together to find the air column length that gives the loudest sound. Measure the distance from the top of the resonance tube to the water level.

Length of the resonance tube = ______________ m

4) Record the diameter of the resonance tube.

Diameter of the resonance tube = ____________ m

5) Make a corrected length by adding 0.4 times the diameter of the tube to the measured length of the air column. This length accounts for the small amount of air just above the tube that also vibrates.

Corrected Length = ______________m

6) The corrected length is one-fourth of the wavelength of the sound vibrating in the air column. Record the wavelength.

Wavelength = ______________m

7) Determine the velocity of sound and record.

Speed of sound in air = __________m/s.

8) Repeat each of the above steps for a second tuning fork:

f = ______________ Hz

Length of the resonance tube = ______________ m

Corrected Length = ______________ m

Wavelength = ______________ m

Speed of sound in air = ______________ m/s.

The accepted value for the speed of sound in air is 332m/s at 0C. the speed of sound in air increases by 0.6m/s for each degree C above zero. Compute the accepted value for the speed of sound at room temperature.

Calculate the percentage difference between this and the average of your two measured values.

Part II: Speed of Sound in a metal

1. Ensure that the cork dust in the glass tube of the Kundt apparatus is evenly distributed throughout its length (remove from the support if necessary).
2. Record the rod material and measure its length.

   material:____________ length_____________ m
3. Adjust the position of the rod so that it may be clamped at its midpoint being careful that the disk on the end of the rod does not touch the walls of the glass tube leaving it free to vibrate.
4. Rub a little resin on a cloth and stroke from about six inches from the end to off the end. If the cork dust is not agitated, change the length of the air column by moving the glass tube a short distance. Continue this procedure until the best resonance is obtained. (Rosin has been used to replace cork dust in some of the tubes. In these cases, you may need to look very carefully to see the wave formed)
5. Omit the cork dust loop nearest the disk and measure the distance across all other loops. Record the number of loops included in the measurement. (two piles of dust per wavelength)

   Distance: ___________ m Number of loops: ____________
6. Using the calculated value for the velocity of sound in air from Part I and the average wavelength of the sound in the air column, determine the frequency of sound ($f = v/\lambda$) in the air column. This is the same as the frequency of sound in the metal rod.

   frequency of Sound in air: ________________ Hz
7. Since you know that $f_{air}=f_{rod}$, and that $f=v/\lambda$, you can find the velocity of sound in the metal. The wavelength in the metal is twice the length of the rod since antinodes are set up at each end and the middle of the rod is fixed.

Write an expression for v_{metal}.

8. Calculate v_{metal} from your measured quantities. Compare this with values from the list below by calculating a percentage difference.

material	speed of sound(m/s)
Aluminum	5104
Brass	3500
Copper	3560
Iron	5130
Steel	5000

Don't forget to summarize.

Chapter 7: Optics

33. Optics: Reflection and Refraction

Introduction

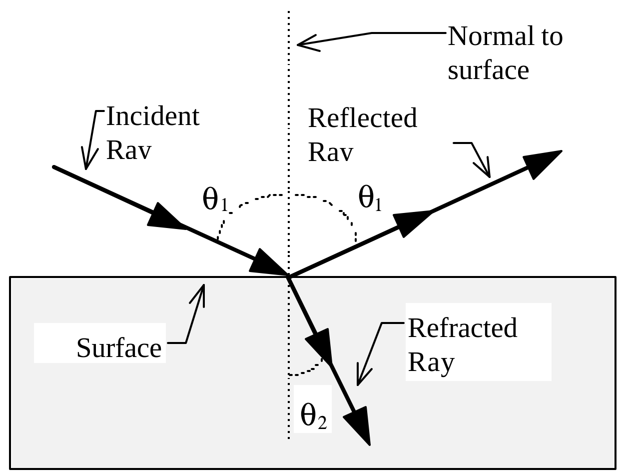
In this lab you will investigate the reflection and refraction of light. *Reflection* of light from a surface is what allows us to see objects that are not luminous. Different colored objects reflect different wavelengths of light. A smooth surface can reflect light in such a way that we can view images of objects. *Refraction* occurs when light passes from one medium to another and is bent. For example, a straw in a glass of water appears to be bent because the light itself bends when traveling between water, glass, and air.

Equipment

HeNe laser	plastic half circle, wedge	water	glass slabs
poster board	wooden blocks (video boxes)	protractor	meter stick
masking tape	straight edge or ruler	beakers for fetching water	

prism, paper cardboard slit and white light source (overhead) set up as demonstration

Theory



When light is reflected from a smooth surface the angle of reflection is equal to the angle of incidence (Law of Reflection):

$$q_{incident} = q_{reflected}$$

When light passes through a surface from one medium to another (air to water for example) the angles obey *Snell's Law:*

$$n_1 \sin q_1 = n_2 \sin q_2$$

where n_1 and n_2 are each the *index of refraction* of the two media. The index of refraction is a measure of the way in which the light interacts with the medium. The speed of light in the medium is given by $v = c/n$ and the wavelength in the medium by $l = l_0/n$, where l_0 is the wavelength in vacuum and c is the speed of light in vacuum. The frequency of light does not change when traveling from one medium to another. The index of refraction is always a number greater than or equal to one. The index of refraction will depend on wavelength.

Index of refraction at l =589 nm					
Air	1.000293	Glass, crown	1.52	Polystyrene	1.49
Water	1.333	Glass, flint	1.66	Diamond	2.419

Procedure

Reflection and Refraction with a HeNe laser

Obey all instructions regarding laser safety: you do not want to shine the laser in anyone's eye!

Turn off the laser when you are not using it

The direct beam is most hazardous: always make a "block" out of poster board and block the laser beam before it leaves the lab table.

The reflected beam is less dangerous but should still be watched. Keep track of reflections and make sure they do not shine in anyone's eye.

Reflection of light:

Equipment needed: HeNe laser, glass plate, ruler, protracter, paper, beam blocks

For this experiment you will look at reflection of a laser from a piece of plate glass.

Tape a piece of paper on a level surface (such as the table) and draw a line at the center to use as the base line for the position of your "mirror"

Set up the HeNe laser so that the beam is reflected from the glass straight back at the laser. (If your table is not exactly level the beam may be high or low when it gets back to the laser). *Obey laser safety instructions and make sure the transmitted beam is blocked.*

Hold a straight edge vertically down from the laser beam (let the laser beam graze the edge of the ruler) and sketch a line on the paper to trace the path of the incident beam.

Measure the angle between the glass and the beam and confirm that it is 90 degrees. (Does this agree with the law of refraction?)

Make sure the laser beam is lined up with the line you drew and *rotate the glass* an angle of 30 degrees from its original position. This makes the angle between the laser beam and the *normal* to the glass also 30 degrees.

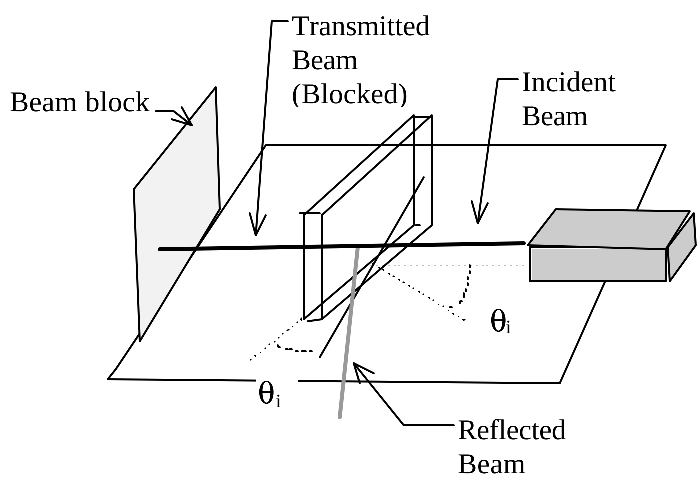


Figure 1: Reflection from a glass

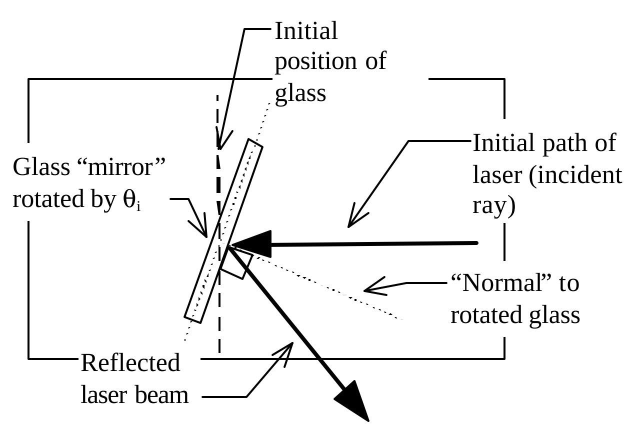


Figure 2: Lines drawn on paper during measurements. The incident and reflected angles are measured from the normal.

On the paper mark the line parallel to the surface of the glass. Mark the line under the reflected laser beam.

Remove the paper. Draw the line *normal* (90 degrees) to the surface of the mirror. Label all these lines clearly.

Measure and label the *angle of incidence* and the *angle of reflection* (these lines are measured from the normal to the surface.)

Repeat for angles of 45 degrees and 60 degrees. (For the final report each member may keep one picture or trace a picture and write their own labels).

Make a table summarizing your data and discuss whether it agrees with the law of reflection.

Refraction of light:

Equipment needed: half-circle plastic dish, wooden block, laser, ruler, paper.

Set up the half circle plastic dish (open side up) similarly to the mirror with the flat side towards the laser. Line up the dish so its flat surface is normal to the laser beam and mark the path of the laser, and the flat surface of the plastic. These lines should be 90 degrees from each other. Make sure the transmitted beam is blocked.

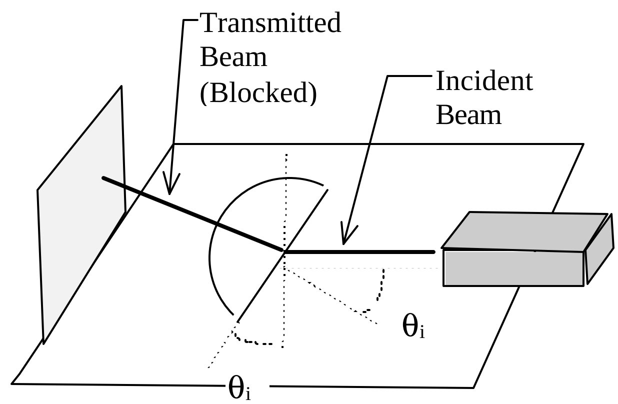


Figure 3: Refraction of a laser beam in water.

Leave the laser fixed and rotate the dish by 30 degrees and mark the new position of the flat side. Notice if anything happens to the transmitted beam.

Fill the dish with water and notice if anything happens to the transmitted laser beam.

The laser beam should be visible in the water (add a little chalk dust if needed). Position the protracter on top of the plastic dish and measure the angle of the refracted beam from the flat side of the dish.

Remove the paper and on it draw the line for the refracted beam using the angle you measured. Label all lines clearly.

Repeat for angles of 46 degrees and 60 degrees. Each lab partner should have a sketch which they have labeled for their final report.

Make a table summarizing your data (angle of incidence and angle of refraction).

In a third column in your table calculate the angle of refraction using Snell's Law and the percent difference from your experimental observation. Discuss any discrepancies.

Refraction from water to air (Internal reflection):

Equipment needed: wedge shaped plastic dish, wooden block, laser, ruler, paper.

Line up the wedge shaped dish so that the laser refracts through the wedge, as shown. Find the transmitted beam. Fill the wedge with water and observe what happens to the transmitted beam.

You will be looking at reflection from the second surface (inside the water). If necessary, adjust the angle of the "prism" until you can see the transmitted beam.

Now q_1 is the angle inside the water and q_2 is the angle in the air as the beam leaves the second surface of the prism. Which is bigger q_1 or q_2?

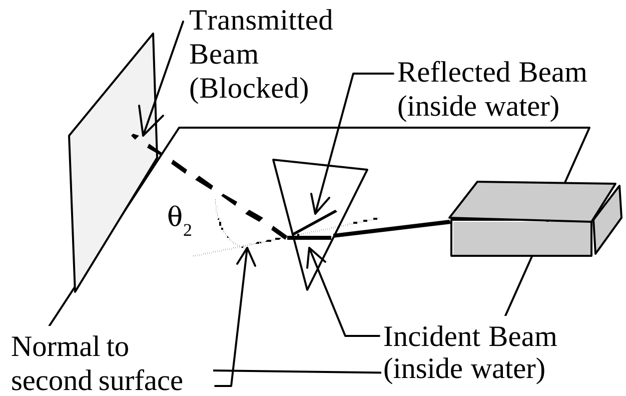


Figure 4: Internal reflection inside the wedge

In the water you should also see *internal reflection* on the second surface (the water to air interface.) Rotate the prism and observe the intensity of both the reflected and transmitted beams. How does the intensity of the transmitted light and of the internal reflection change with angle?

In *total internal reflection* there is no transmitted light. Can you observe this? Is the angle from the normal of the incident beam (hitting the second surface from inside the water) large or small in this case?(Compared to when there is transmitted light)

Refraction by a prism:

Equipment needed: white light source, prism

Set up a white light source and prism so that the prism disperses the light. Make a sketch of what you see, noting the colors.

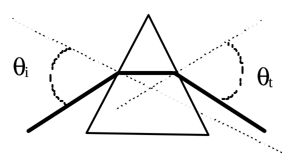


Figure 5: Refraction by a prism

Figure 5 shows a single color of light being refracted by a 60 degree (equilateral triangle) prism. Because the prism (glass) has an index of refraction greater than air the light is bent inward when passing from air to glass. Different wavelengths (colors) will have different indexes of refraction in glass but nearly the same index of refraction in air. According to your observations, is blue light be refracted more or less than red light by the prism? Is the index of refraction greater or lesser for blue light than for red light?

Index of refraction of glass:

Equipment needed: glass slabs, wooden block, laser, meter stick and supports.

Set up the laser so that it hits a meter stick (positioned on metal supports). Mark a line under the laser for the undeflected position of the beam.

In the laser beam place several slabs of glass positioned at a 45 degree angle (measure carefully). Observe what happens to the transmitted beam as you insert the glass.

Is the laser being deflected at an angle (in which case the distance of deflection gets larger as you move farther from the glass) or is it simply displaced a constant distance (in which case it travels parallel to the undeflected path but displaced to the side)? Use a card to trace the laser beam path until you understand what is happening.

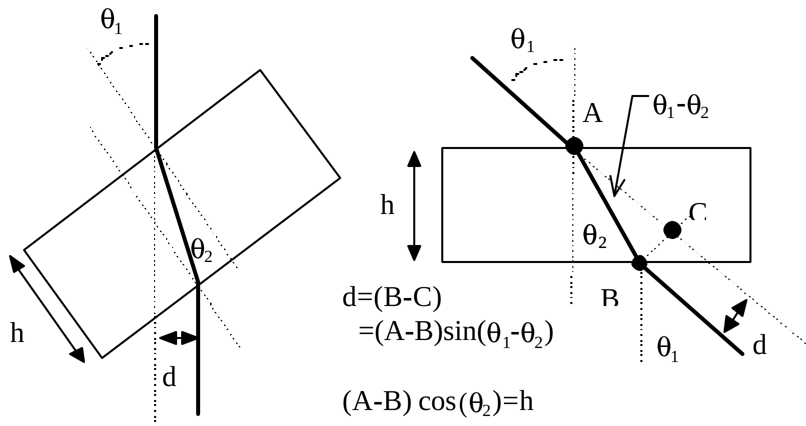


Figure 6: Refraction in glass (two views). For a slab of glass with parallel sides the emerging angle is equal to the incident angle and the beam of light is simply displaced a distance d.

Measure the distance of deflection and the total width of the glass slabs. Make sure you have a careful measure of the angle of incidence on the glass slabs.

Make a sketch of the path of the laser beam through the glass and label the angle of incidence, the thickness and the defection distance.

Use Snell's Law, your total glass thickness, h, the incident angle, q_1, and a value of n=1.5 to calculate the deflection distance, d, through the glass. Does your glass have an index of refraction greater or less than 1.5? (Does the deflection increase or decrease with index of refraction?)

34. Optics: Lenses and Mirrors

Introduction

Mirrors reflect light while lenses transmit light. Both can form images, however. In this lab you will study spherical lenses and mirrors. A spherical surface is simply some portion of the surface of a sphere. It is characterized by its *radius of curvature, R.*.

Equipment

Meter stick optical "bench"	Light source, object card	Spherical and cylindrical mirrors	Screen (white cardboard)
Lens holders	Convex lenses (f = 10-30 cm)	Concave lens (f=-10 to -30 cm)	Ruler for ray diagrams.

Theory

A spherical mirror is a section of a sphere. If the "shiny" side is inside the sphere it is a *concave* surface (Figure 7). If the mirror is on the outside surface then it is *convex* (flexed out). As with all mirrors, light reflecting from the surface obeys the law of reflection: the reflected angle is equal to the incident angle. The normal to a spherical surface passes through its radius. From this it can be shown that rays parallel to the optic axis of a concave spherical mirror will all be reflected towards the focal point (converging). Rays reflected from a convex spherical mirror will diverge away from a focal point behind the surface. (See Figure 8)

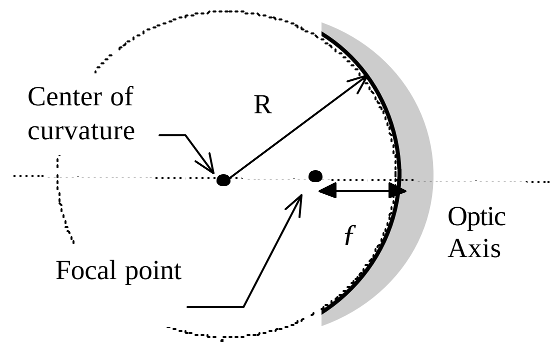


Figure 7: A concave spherical mirror.

The *focal length* (distance to focal point) of a spherical mirror is given by

$$\frac{1}{f} = \frac{2}{R} \qquad \text{or} \qquad f = \frac{R}{2}.$$

Lenses form images by refracting light. A spherical lens can be made of any transparent material with a curved surface and an index of refraction that differs from the medium its in. For a lens with index of refraction n and spherical surface(s) of radius R_1 and R_2 which is placed in air, the focal length is given by

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

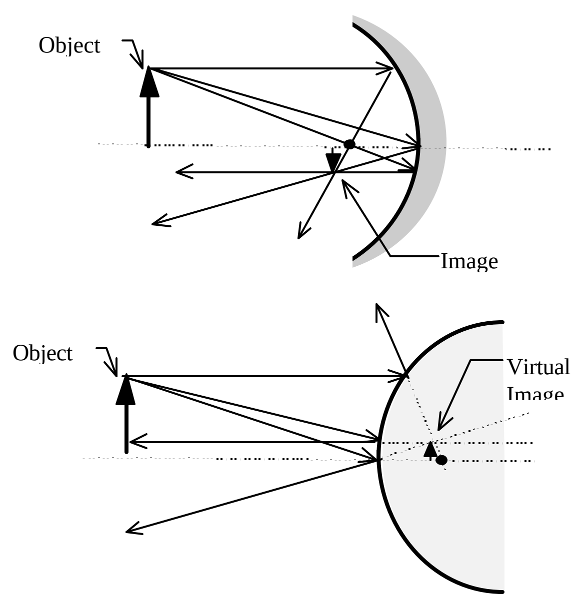


Figure 8: Concave converging mirror (top) and convex diverging mirror (bottom).

For a spherical lens, parallel light incoming to a lens with convex surfaces will converge on its focal point, while parallel light incident on a concave lens will diverge away from its focal point. (Figure 9) The *image location* for either mirrors or lenses can be found by a *ray diagram*. A ray diagram is constructed using the following rules for a converging (or diverging) lens:

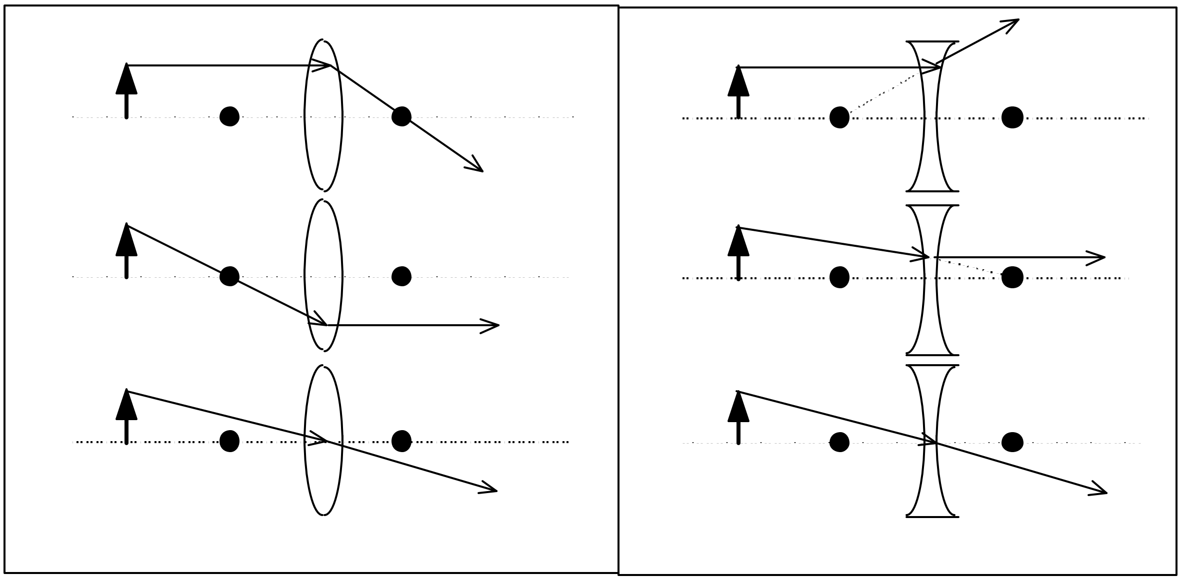


Figure 9: Converging (left) and diverging (right) lenses

Parallel incoming rays will be bent towards (or away from) the focal point.
A ray incoming from (or towards) the focal point will be refracted so it is parallel.
A ray through the center of a lens is unchanged.
These rays are shown in Figure 9. A real image is formed if these rays intersect. If the diverge when exiting the lens, a virtual image is found by tracing them back behind the lens. Similar rules apply to mirrors as shown in Figure 8.

The *magnification* of an image is the ratio of image height (*h'*) to object height (*h*). Drawing similar triangles shows that this is equal to the ratio of image distance, *i* to object distance, *p*. The relationship between image distance, object distance and focal length is given by

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f} \qquad \text{and the magnification by} \qquad m=\frac{h'}{h}=-\frac{i}{p}$$

A negative magnification indicates an inverted image. Object distances are positive when on the incoming side of the lens (or mirror). Image distances are positive on the opposite side. Focal lengths are positive for converging lenses and negative for diverging lenses.

Procedure

Part A: Spherical Mirrors.

Finding the focal length and radius.

Light from an object located very far away is nearly parallel. This can be used to find the focus of a spherical mirror. Take a mirror and a white card to the window. Hold the mirror up to the light from the sun and adjust the position of the white card until the light is focused on it. (If it is night time use another light source far from the mirror). Measure the distance from the mirror to the card.

Have a second person repeat the measurement to check your accuracy. Average the results.

Calculate the radius of curvature of the mirror. Does your answer seem reasonable? Draw a ray diagram for the mirror showing the incoming light coming in parallel and converging on the focus.

Images in spherical and cylindrical mirrors.

Hold one hand up next to your face. Hold the spherical mirror so that this hand is farther than the focus. Tilt the mirror til you see your hand. Describe what you see. Is the image upright or inverted? Smaller or larger?

Make a ray diagram for a spherical converging mirror and an object farther than the focus. Explain whether or not your diagram agrees with your observations.

Repeat these two steps using the convex side of the mirror as a diverging mirror. (You can look directly at your face instead of your hand now).

Look at the images in a flat mirror and a cylindrical mirror. What do you see? How does it compare to the spherical mirror? Explain why the image in the cylindrical mirror looks the way it does.

Part B: Spherical Lenses

Here you will set up the lenses on an optical bench and find the images of the light through a triangular aperture. First try to use a screen to find the image. If you cannot do this it may be a virtual image. In this case you can look through the lens to see the image but cannot actually measure the image distance. In each case you should record the relative size and orientation of the image.

Convex (Converging) Lenses

Measure image distances and observe the size and orientation of the image for the following cases using a +20 cm focal length lens: object distance = 5 cm, 10 cm, 25 cm, and 40 cm.

For the 10 cm and 25 cm object distances, draw a ray diagram (label everything in your diagram to explain what it is). The diagrams should be to scale and drawn accurately with a ruler. For each case compare your results to the ray diagrams.

Measure the image distances and magnifications for an object distance of 40 cm and lenses of focal lengths +10 cm, +20 cm (done above) and +30 cm.

For each case calculate the theoretical image distance and magnification and compare to your result.

Concave (Diverging) Lenses

Using a -20 cm focal length lens, find the images for object distances of +10 cm and +20 cm.

Record your observations, draw ray diagrams and do calculations as described above.

Part C: Making a telescope

A telescope magnifies images which are very far away.

A terrestrial telescope forms an upright image: Rays from the distant object are nearly parallel and are focused by a converging lens (called the Objective). Because the rays are nearly parallel, the image from the Objective is nearly at its focus. A diverging lens (called the Eyepiece) is placed inside the focal distance. The image formed by the Objective becomes the object for the Eyepiece which then forms a much bigger, upright virtual image which can be viewed through the Eyepiece. If the focal point of the Eyepiece is near the focal point of the Objective then the final image should be very large.

A astronomical telescope forms an inverted image: Again a converging Objective lens focuses rays from very distant object. In this case a converging (called the Eyepiece or Occular) is placed outside the focal distance of the Objective and and is used to form a final inverted virtual image that is magnified.

Construct a telescope. Describe it for your report.

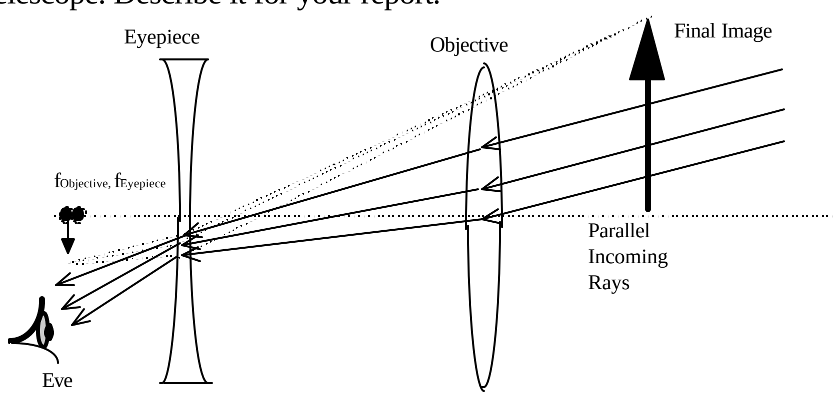


Figure 10: Terrestrial (Galilean) Telescope (not to scale)

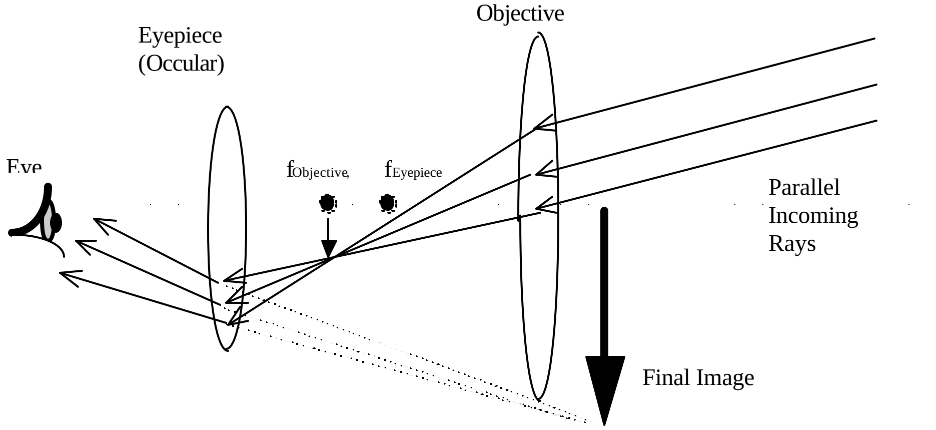


Figure 11: Astronomical telescope (not to scale)

Part A: Spherical Mirrors.

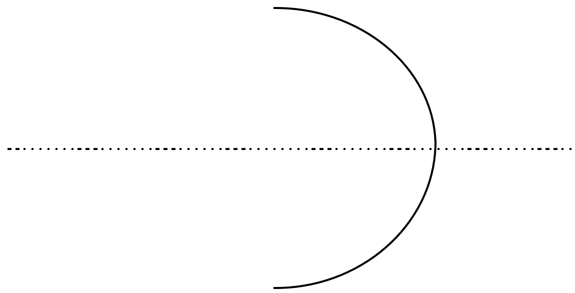
Distance from mirror to screen when focused ______________________

____

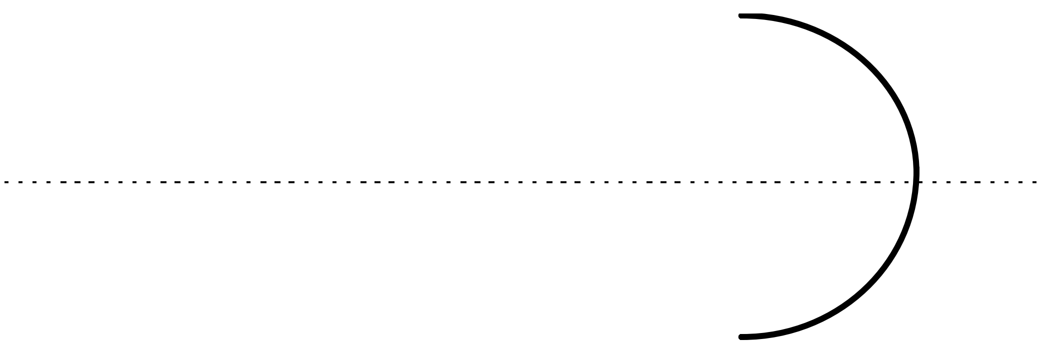
Average: ______________________

Radius (calculated) ______________________

Ray Diagram (Draw accurately to scale and show several incoming rays, reflected rays, the focal point and the radius.



Images in Mirrors (USE THIS AS A TEMPLATE FOR ALL PARTS)

Converging Mirror, Object farther than focus:
Description of image (what your hand looks like!)
Ray Diagram (draw to scale using a ruler for straight lines!) 
Does the diagram agree with what you saw? **EXPLAIN!**

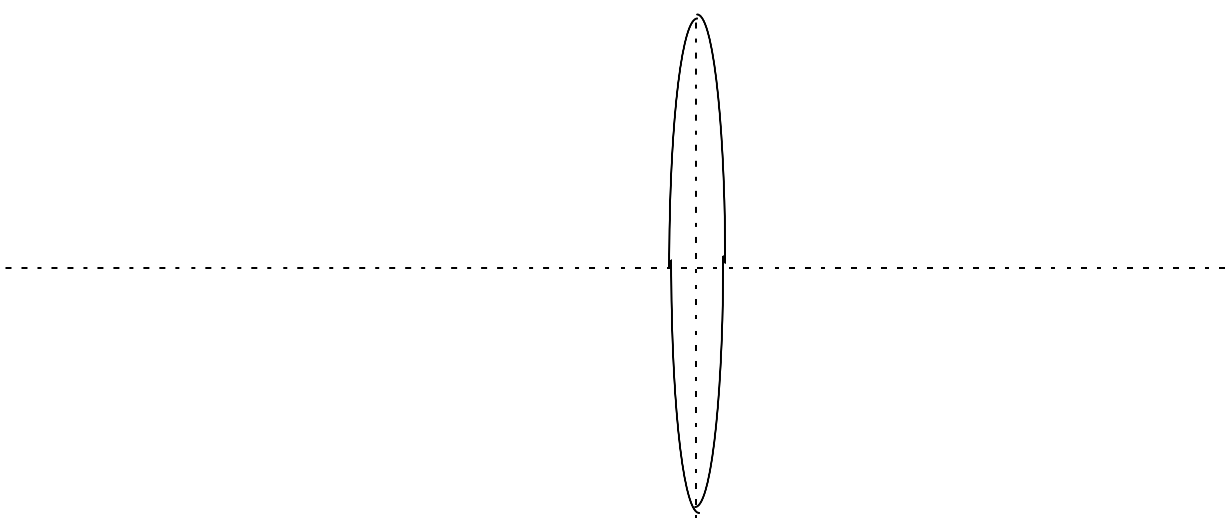
Part B: Spherical Lenses

USE THIS AS A TEMPLATE FOR EACH CASE Follow with discussion.

Converging Lens: Data and Calculations

Focal Length, *f*	Object Distance, *p*	Image Distance, *i*	Describe Relative Image Size	Image Orientation
Theoretical Image Distance			Theoretical Magnification, *m*	

Ray Diagram: Use a ruler and make the drawing to scale. Mark the focal points and label all rays.



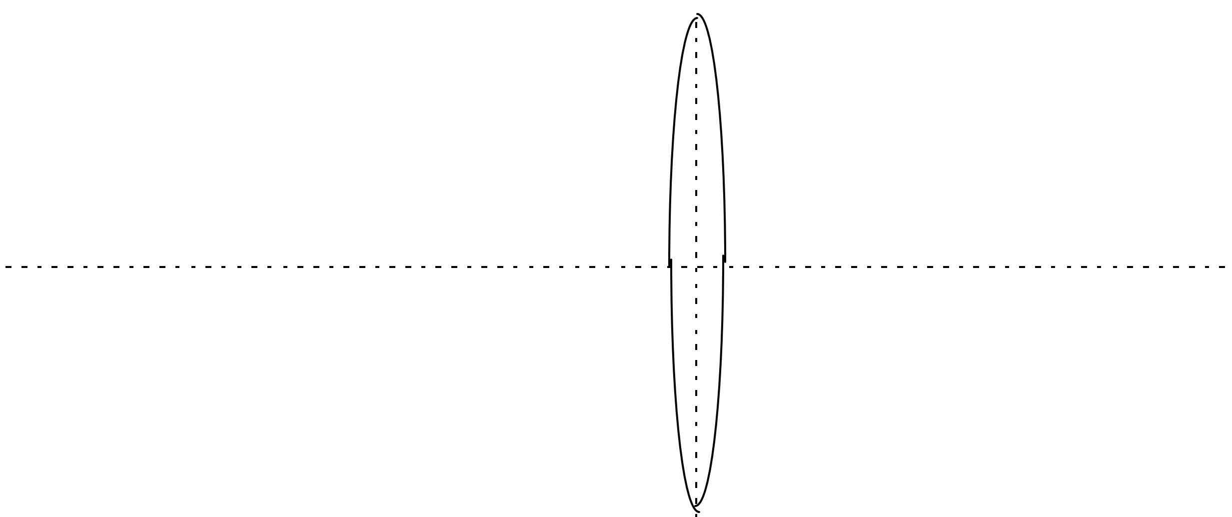
Part B: Spherical Lenses

USE THIS AS A TEMPLATE FOR EACH CASE Follow with discussion.

Converging Lens: Data and Calculations

<table>
<tr><td>Focal Length, f</td><td>Object Distance, p</td><td>Image Distance, i</td><td colspan="2">Describe Relative Image Size</td><td colspan="2">Image Orientation</td></tr>
<tr><td></td><td></td><td></td><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td colspan="2">Theoretical Image Distance</td><td colspan="2"></td><td colspan="2">Theoretical Magnification, m</td><td></td></tr>
</table>

Ray Diagram: Use a ruler and make the drawing to scale. Mark the focal points and label all rays.



Part B: Spherical Lenses

USE THIS AS A TEMPLATE FOR EACH CASE Follow with discussion.

Diverging lens: Data and Calculations

<table>
<tr><td>Focal Length, f</td><td>Object Distance, p</td><td>Image Distance, i</td><td colspan="2">Describe Relative Image Size</td><td colspan="2">Image Orientation</td></tr>
<tr><td></td><td></td><td></td><td colspan="2"></td><td colspan="2"></td></tr>
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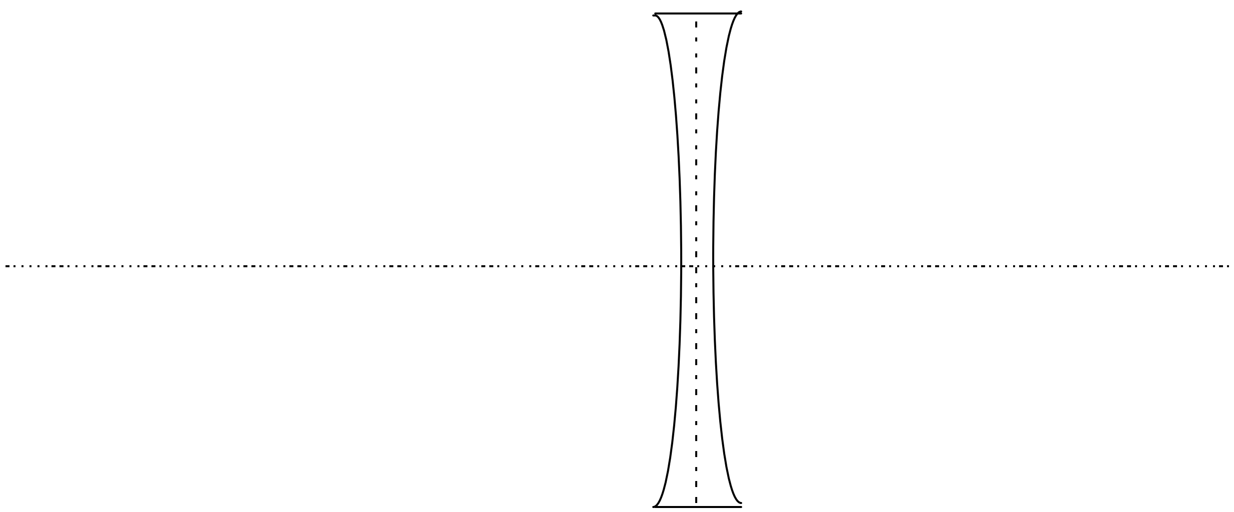
Ray Diagram: Use a ruler and make the drawing to scale. Mark the focal points and label all rays.



Part B: Spherical Lenses

USE THIS AS A TEMPLATE FOR EACH CASE Follow with discussion.

Diverging lens: Data and Calculations

<table>
<tr><td>Focal Length, f</td><td>Object Distance, p</td><td>Image Distance, i</td><td colspan="2">Describe Relative Image Size</td><td colspan="2">Image Orientation</td></tr>
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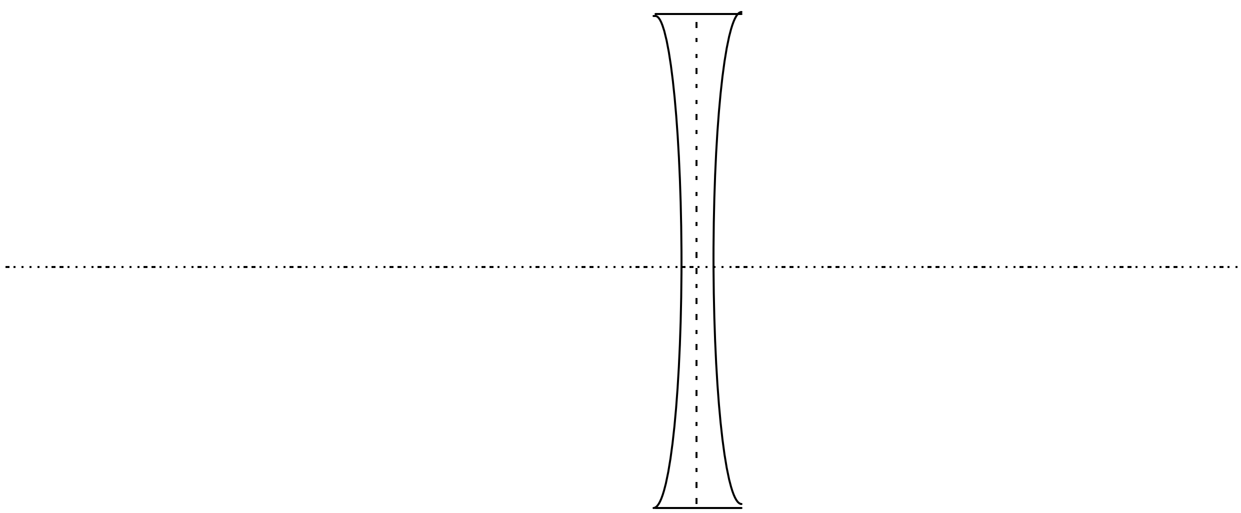
</table>

Ray Diagram: Use a ruler and make the drawing to scale. Mark the focal points and label all rays.



Data Summary

Data and Calculations

Focal Length, *f*	Object Distance, *p*	Image Distance, *i*	Relative Image Size	Image Orientation	Theoretical Image Distance	Theoretical Magnification

35. Wave Optics: Interference and Diffraction

Introduction

In our optics labs we have described the properties of light simply in terms of rays, using the laws of reflection and refraction. This description works well for many phenomena, including image formation with lenses and mirrors, but for other circumstances it fails. In this lab we examine some phenomena which clearly require a different description. This alternative description is called wave optics, to distinguish it from the ray description which is called geometric optics.

Equipment

HeNe Laser	diffraction slits	meter stick	tape
poster board	hologram set up as a demonstration		

Obey all instructions regarding laser safety: you do not want to shine the laser in anyone's eye!

Turn off the laser when you are not using it

The direct beam is most hazardous: always make a "block" out of poster board and block the laser beam before it leaves the lab table.

The reflected beam is less dangerous but should still be watched. Keep track of reflections and make sure they do not shine in anyone's eye.

Theory

If the waves have the same sign (are *in phase*) then the two waves constructively interfere, the net amplitude is large and the light intensity is strong at that point. If they have opposite signs, however, they are *out of phase* and the two waves destructively interfere: the net amplitude is small and the light intensity is weak. It is these areas of strong and weak intensity which make up the interference patterns we will observe in this experiment.

Interference can be seen when light from a single source arrives at a point on a viewing screen by more than one path. Because the number of oscillations of the electric field (wavelengths) differs for paths of different lengths, the electromagnetic waves can arrive at the viewing screen with a *phase difference* between their electromagnetic fields. If the Electric fields have the same sign then they add *constructively* and increase the intensity of light, if the Electric fields have opposite signs they add *destructively* and the light intensity decreases.

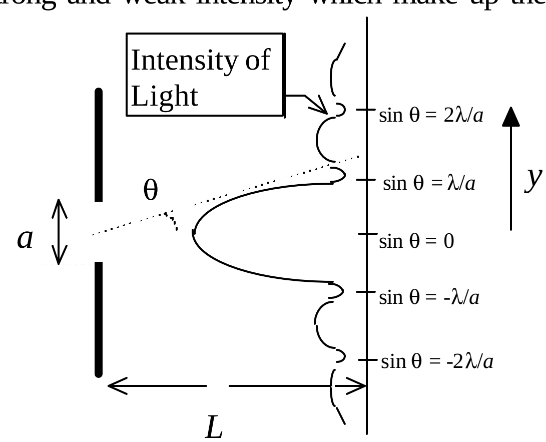


Figure 12: Diffraction by a slit of width *a*. Graph shows intensity of light on a screen.

<u>Diffraction</u> can be observed when light travels through a hole (in the lab it is usually a vertical *slit*) whose width, a, is small. Light from different points across the width of the slit will take paths of

different lengths to arrive at a viewing screen (Figure 12). When the light interferes destructively, intensity minima appear on the screen. Figure 12 shows such a diffraction pattern, where the intensity of light is shown as a graph placed along the screen. For a rectangular slit it can be shown that the minima in the intensity pattern fit the formula

$$a \sin q = m l \ ,$$

where m is an integer $(\pm 1, \pm 2, \pm 3, \ldots)$, a is the width of the slit, l is the wavelength of the light and q is the angle to the position on the screen. The m^{th} spot on the screen is called the m^{th} order minimum. Diffraction patterns for other shapes of holes are more complex but also result from the same principles of interference.

<u>Two-slit Interference:</u> When laser light shines through two closely spaced parallel slits (Figure 13) each slit produces a diffraction pattern. When these patterns overlap, they also interfere with each other. We can predict whether the interference will be constructive (a bright spot) or destructive (a dark spot) by determining the path difference in traveling from each slit to a given spot on the screen. Intensity maxima occur when the light arrives *in phase* with an integer number of wavelength differences for the two paths

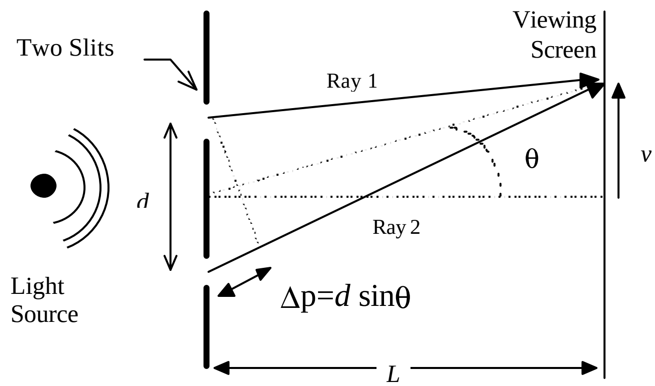


Figure 13: Interference of light from two slits. A maximum occurs when $\Delta p = m l$ and a minimum when $\Delta p = (m + 1/2) l$, where *m*=0,1,2,…

$$\Delta p = d \sin q = m l \qquad \text{where} \qquad m = 0, \pm 1, \pm 2, \pm 3, \ldots$$

and the interference will be destructive if the path difference is a half-integer number of wavelengths so that the waves from each slit arrive *out of phase* with opposite signs for the electric field.

$$\Delta p = d \sin q = \left(m + \frac{1}{2} \right) l \qquad \text{where} \qquad m = 0, \pm 1, \pm 2, \pm 3, \ldots$$

<u>Small Angle Approximation</u> The formulae given above are derived using the *small angle approximation*. For small angles q (given in *radians)* it is a good approximation to say that

$$q \approx \sin q \approx \tan q \qquad \text{(for } q \text{ in radians)}$$

For the figures shown above this means that $q \approx \sin q \approx \tan q = \dfrac{y}{L}$

Procedure

Part A: Diffraction

The diffraction plate has slits etched on it of different widths and separations. For this part use the area where there is only a single slit.

Look through the slit (holding it very close to your eye). See if you can see the effects of diffraction.

Set the laser on the table and aim it at the viewing screen. **DO NOT LOOK DIRECTLY INTO THE LASER OR AIM IT AT ANYONE! DO NOT LET REFLECTIONS BOUNCE AROUND THE ROOM.**

For two sizes of slits, examine the patterns formed by single slits. Set up the slit in front of the laser. Record the distance from the slit to the screen, L. For each of the slits, measure and record a value for y on the viewing screen corresponding to the center of a dark region. Record as many distances, y, for different values of m as you can. Use the largest two or three values for m which you are able to observe to find a value for a. The HeNe laser has a wavelength of 633 nm.

Pull a hair from your head. Mount it vertically in front of the laser using a piece of tape. Place the hair in front of the laser and observe the diffraction around the hair. Use the formula above to estimate the thickness of the hair, a. (The hair is not a slit but light diffracts around its edges in a similar fashion.) Repeat with observations of your lab partners' hair.

Part B: Two-slit Interference

Using the two-slit templates, observe the patterns projected on the viewing screen. Observe how the pattern changes with changing slit width and/or spacing.

For each set of slits, determine the spacing between the slits by measuring the distances between minima on the screen. (The smaller spacings give are from the two slits patterns interfering, if they get too small to measure accurately, just make your best estimate.) You will need to record distances on the screen y and the distance from the slits to the screen, L. Answer the questions.

Part C: Viewing a hologram

A hologram records a three-dimensional image because it records a virtual image that contains both amplitude and phase information. When looking at a three dimensional object, both the amplitude and phase of light vary as you look at the object from different angles. Since a hologram, unlike an ordinary photograph, "saves" the phase information, the image of the hologram changes with viewing angle.

Part A: Diffraction

Slit Used

Sketch:

L= ________________

l = ________________

Diffraction Order, m	Distance, y	y / L	Angle q in *radians*	sin q	a $\left(= \frac{m\,l}{\sin q}\right)$

Calculation of slit width (show your work):

Slit width = ______________________________

Slit Used

Sketch:

L= ________________

l = ________________

Diffraction Order, m	Distance, y	y / L	Angle q in *radians*	sin q	a

Calculation of slit width (show your work):

Slit width = ______________________________

Estimate of width of hair (Describe your measurements, show your work):

Questions:

How does the distance between diffraction minima change (increase or decrease) as the slit width is decreased?

Why don't you see diffraction patterns when light is sent through a large hole?

Part B: Two Slit Interference

Slits Used

Sketch:

L= ________________

l = ________________

Diffraction Order, m	Distance, y to minimum	y/L $= \tan q \approx \sin q$	$\left(m+\frac{1}{2}\right)l$ $= \Delta p$	d = $\frac{\Delta p}{\sin q}$

Slits Used

Sketch:

L= ________________

l = ________________

Diffraction Order, m	Distance, y to minimum	y/L $= \tan q \approx \sin q$	$\left(m+\frac{1}{2}\right)l$ $= \Delta p$	d = $\frac{\Delta p}{\sin q}$

Questions:

Is a more complex (more intricate structure) interference pattern observed with closely spaced slits or with widely separated slits?

Why is that so?

Appendices

Appendix A: Percent Difference

When comparing an experimental value to a theoretical value it is usually more useful to consider the percent difference rather than the absolute numerical difference. The percent difference is give by

$$\%\,\text{Difference} = \frac{Experimental - Theoretical}{Theoretical} \times 100\%$$

The percent difference will be positive if the experimental value is greater than theoretical and negative if it is less than the theoretical value. The theoretical value is assumed to be well-known, which is why it is in the denominator.

When comparing two experimental values we may not know which is more accurate. In this case we need to calculate the average

$$Average = (ExperimentalOne + ExperimentalTwo)/2$$

and then we can find the percent difference between the two values by dividing by the average:

$$\%\,\text{Difference} = \frac{ExperimentalOne - ExperimentalTwo}{Average} \times 100\%$$

This does not "favor" either experimental value. The sign in this case is not meaningful since we are not sure which experimental value might be more accurate.

Appendix B: Uncertainties and significant figures

All measured values should be reported as $x \pm \Delta x$, where x is the measured (or calculated) value and Δx is the uncertainty. (Other symbols used for uncertainty include s_x and $d\,x$). The ratio $\Delta x/x$ is defined as the fractional uncertainty and $100\% \times \Delta x/x$ is the percent uncertainty.

In this course, uncertainties will be rounded to one significant figure. The last significant figure in the value x should be in the same decimal position as the uncertainty. For example, if you measure the length of a metal rod using a ruler which has mm markings, you may be able to measure the length to within 0.5 mm. The length would be reported as $37.0\text{mm} \pm 0.5\text{mm}$ or perhaps as $3..70\text{cm} \pm 0.05\text{cm}$. Notice that the last significant figure in the data is the same size as the uncertainty.

Random uncertainties

Random uncertainties are inherent in the process of measurement. They can be reduced by making careful and/or repeated measurements. Uncertainty can be estimated by looking at how “scattered” the data is about the mean (average). For large numbers of measurements a more sophisticated estimate called the “standard deviation” is calculated.

If a measurement is made only once, the random uncertainty must be estimated based on how precisely you were able to read the measurement off of the provided scale. Often, this uncertainty is quoted as being one-half of the smallest division on the measuring device, as in the example above.

The standard deviation

If the measurement is repeated several times, the random uncertainty can be calculated statistically. Consider a measurement of a quantity, x, made N times, providing the measurements $x_1, x_2, x_3, \cdots x_N$. From these measurements, we report

$$\bar{x} = \frac{1}{N} \cdot \left(x_1 + x_2 + \cdots + x_N \right) = \frac{1}{N} \sum_{i=1}^{N} x_i \qquad \text{and} \qquad s_x = \frac{1}{\sqrt{(N-1)}} \times \sqrt{\sum_{i=1}^{N} \left(x_i - \bar{x} \right)^2}$$

where $\bar{x}$ is the mean (also called the average) and s_x is an estimate of the uncertainty called the "standard deviation of the mean".

Systematic uncertainties

Systematic uncertainties are inherent in the instruments that we use to obtain data. Accurately calibrated instruments can reduce it. Systematic uncertainties are often provided by the manufacturer of the equipment. I will give you these uncertainties when necessary. The systematic and random uncertainties for a given measurement are summed, such that $\Delta x = \Delta x_{random} + \Delta x_{systematic}$.

"Propogation" of uncertainty

When several measured data values are used to calculate a final value, the uncertainties associated with the experimental measurements must be combined to find the final uncertainty

Addition and subtraction

When adding or subtracting two measured values, the uncertainties are added.

Example:

If $c = a + b$ or $c = a - b$ then:

Addition $\begin{cases} c = (a \pm \Delta a) + (b \pm \Delta b) \Rightarrow c = a + b \pm (\Delta a + \Delta b) \\ \quad \text{or} \quad c = a + b \quad \text{and} \quad \Delta \text{c} = (\Delta a + \Delta b) \end{cases}$:

Subtraction $\begin{cases} c = (a \pm \Delta a) - (b \pm \Delta b) \Rightarrow c = a - b \pm (\Delta a + \Delta b) \\ \quad \text{or} \quad c = a - b \quad \text{and} \quad \Delta \text{c} = (\Delta a + \Delta b) \end{cases}$

Multiplication and division

When multiplying or dividing two measured values, the *fractional* uncertainties are added.

If $c = a \times b$ or $c = a/b$ then:

Multiplication $\begin{cases} c = (a \pm \Delta a) \times (b \pm \Delta b) \Rightarrow c = a \times b \pm a \times b \times \left(\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b} \right) \\ \quad \text{or} \quad c = a \times b \quad \text{and} \quad \dfrac{\Delta c}{c} = \dfrac{\Delta a}{a} + \dfrac{\Delta b}{b} \end{cases}$

Division
$$\begin{cases} c = (a \pm \Delta a)/(b \pm \Delta b) \Rightarrow c = a/b \pm (a/b) \times \left(\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b} \right) \\ \quad \text{or} \quad c = a/b \quad \text{and} \quad \dfrac{\Delta c}{c} = \dfrac{\Delta a}{a} + \dfrac{\Delta b}{b} \end{cases}$$

Appendix C: Understanding Resistor Codes.

Resistors are often color coded with their values. There are usually four bands of color. The first two bands represent two digits and the third band represents additional powers of ten. The fourth band is either silver or gold, indicating the value is good to 10% or to 5%. The numbers represented are:

Color	Black	Brown	Red	Orange	Yellow	Green	Blue	Violet	Gray	White
Value	0	1	2	3	4	5	6	7	8	9

A resistor with bands of Brown, Blue, Red and Gold would signify the three digits 162 and have a value of 16 x 10^2 = 1600 Ohms.

Appendix D: Reading a Vernier

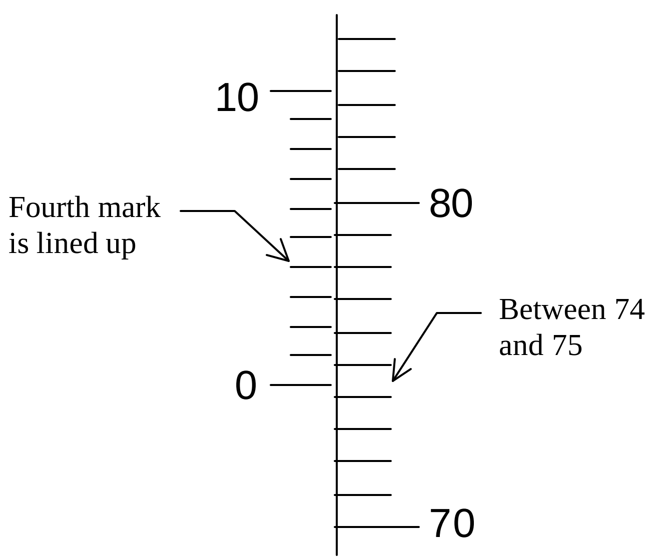


Figure 1: Vernier scales reading 74.4

Vernier scales consist of two scales side by side. One scale gives a rough scale reading and the second determines the decimal reading.

In the figure the zero of the left hand scale is lined up between 74 and 75 on the right hand scale. This tells us the reading is between 74 and 75. To get the exact decimal reading look on the left hand scale to find where a marker lines up with one on the right hand scale -- this is the fourth line, so the reading should be 74.4 .

Appendix E: Using the computer lab for graphing

The computer can be used to graph data and fit a line to your data points. The method used is a "least squares fit", which finds the straight line which best represents the data by minimizing the sum of the distances (squared) from the line to all the points. The programs will also perform other types of fits (quadratic curves etc.) These are a summary of instructions for two programs in the computer lab (You can use either one.)

Kalaidagraph 3.0

1. Open Kalaidagraph. If a data window is not open, select NEW from the FILE menu.
2. Enter data in column. Enter the data for the horizontal axis (e.g., time) in column A and data for the vertical axis (e.g., velocity) in column B.
3. You can let the program calculate numbers. For example, enter " =1/60 " in a cell in order to calculate 1/60.
4. After entering two columns of data, select all the data you want to plot with the mouse.
5. From the GALLARY menu, choose LINEAR and then SCATTER PLOT to make a plot of y versus x.
6. Choose column A for the "x" data and column B for the "y" data.
7. After your graph pops up, you can choose CURVE FIT and then LINEAR to fit a straight line.
8. If a formula for your straight line is not shown make sure that DISPLAY EQUATION is selected in the PLOT menu.
9. Your equation displays as y=-8.3+41.4x R=0.9599. This gives the formula for the line in terms y=b+mx, where b is the intercept and m is the slope. The term R (not part of the formula) is called the remainder and measures how closely the line fits the data.
10. From the FILE menu choose PRINT GRAPHICS to print your graph. Generally the "landscape" orientation will give a better print. Make sure the printer is "BSH Laser IIF".

CricketGraph

1. Open CricketGraph which is in the Graph III folder. If a data window is not visible, select NEW from the FILE menu.
2. Enter data in column. Enter the data for the horizontal axis (e.g., time) in column 1 and data for the vertical axis (e.g., velocity) in column 2.
3. You can let the program calculate numbers. For example, enter " =1/60 " in a cell in order to calculate 1/60.
4. If too many digits are displayed in your numbers, change this in the OPTIONS menu by selecting NUMERIC FORMAT and choosing a format with fewer zeroes after the decimal.
5. After the data is entered, choose NEW GRAPH and select a SCATTER graph.
6. Let the horizontal axis be column 1 and the vertical axis be column 2.
7. From the OPTIONS menu choose CURVE FIT
8. Change the METHOD for the fit to LINEAR
9. The equation for your line is displayed as y=mx+b, where m is the slope and b is the intercept.
10. From the FILE menu choose PAGE SETUP to change your printout style to landscape.
11. From the FILE menu choose PRINT to print your graph. Make sure the printer is "BSH Laser IIF". **If you select PRINT and nothing comes out of the printer but there is no error message, find your instructor . Do not keep trying to print**

Excel

The "Chart Wizard" button (picture of a chart with a magic wand) will guide you through making a graph or chart from data in an Excel spreadsheet. If you click on the button *nothing happens immediately!* You must select an area on the page with your cursor/mouse. After that dialogue boxes will let you select the type of graph you want and the columns to be graphed.

Notes for Formulae in Excel:

1. Formulae start with an =
2. Reference cells by clicking on them as you enter the formula (have someone show you the first time).
3. Multiply numbers with a *
4. Raise numbers to a power with a ^ (for example x^2 is written as x^2)
5. Use plenty of parenthesis to factor.
6. Functions can be found by using the button with the script f_x.
7. A Square Root is given by SQRT(x) or by (x)^0.5.
8. Enter the formula only in the first cell of the column. After that select the first cell, move the mouse to the corner until you get a black crosshair and then drag down to fill the rest of the column or select an area to fill and use FILL DOWN from the Edit menu.